UNIVERSIDAD NACIONAL DE INGENIERIA

Facultad de Ciencias

TESIS

"MODELING AND CONTROL THEORY APPLIED TO THE WINE FERMENTATION PROCESS"

Para optar el grado académico de Maestro en Ciencias en Matemática Aplicada

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To my father for being mi inspiration to never give up in difficult situations in life.

To my mother for theaching me good values and always supporting me.

To my fiancée for being with me in the good and bad times.

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To my adviser Eladio Ocaña for trusting me, supporting me and giving me the opportunity to go to France to do the thesis in a totally different area.

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Abstract

The thesis has two purposes: The first one is to build a mathematical model that better approximates (with respect to the mathematical model in [2]) to the behaviour of the wine fermentation process with the addition of certain amount of nitrogen at some instant of time, based in [2]. This mathematical model is described by mean ordinary differential equations including some parameters which will be identified solving some optimization problems.

The second purpose of the thesis is to study the improved model but introducing some control variables in order to stabilize in an appropriated sense the corresponding dynamical system of the new model. This is a kind of MIMO nonlinear system control problems and will be solved using the tools in [6].

Introduction

One of the most important steps in the wine production is the fermentation process. The fermentation process consists in the bioconversion of glucose into ethanol and other metabolites which give to the wine a part of its organoleptic characteristics (glycerol, organic acid, aromatic compounds, etc). The yeasts are the ones who perform this conversion. The metabolism of yeast is very complex and it a reason that is continuously studied. Aforetime, the wine fermentation was done manually and empirically. Currently, it is automated in big tanks to do wines to big scales, minimizing the time and the energy consumption, this is the major challenges for oenologists. To do that, first we must study the behaviour of the wine fermentation, this behaviour is represented through a mathematical model, in this case are ordinary differential equations. Then, we must control the dynamical system such that the output of the system stabilizes at a desired value (called setpoint), to do that we are going to use the tools of control theory.

The nitrogen addition during the wine fermentation is a oenological condition, that is to say, the oenologists use that in the wine production to accelerate the fermentation. Before 2004, researchers try to do mathematical models with this oenological condition but unfortunately they are poorly adapted. In 2004, Malherbe made a mathematical model with this oenological condition [2] where that works well in some experiments but others do not. So, one of the two main purpose of this thesis is to improve this mathematical model. The experiments for the creation of the mathematical model are realized in a batch reactor [1], it is a closed tank where there are chemical concentrations (yeast, nitrogen, glucose transporter, glucose, ethanol).

Then, this model is transformed in a mathematical model with control (state-space representation) in a tank like the batch reactor but the difference is that it is added concentrations of nitrogen, glucose transporter, glucose and ethanol with certain velocities Q_1, Q_2 and it is removed concentrations of nitrogen, yeast, glucose transporter, sugar and ethanol with certain velocity $Q = Q_1 + Q_2$. The velocities Q_1 and Q_2 are the controls or input. This kind of tank is called continuous stirred tank bioreactor. The other main purpose of this thesis is to control or manipulate the wine fermentation, that is to say, we are going to add and remove concentration in the continuous bioreactor with certain velocities Q_1 and Q_2 such that the sugar S and the rate production VCO_2 stabilizes at the desired values S^* and VCO_2^* respectively. To do that, we are going to use the tools of control theory for MIMO non linear system [6].

We now describe the content of the thesis which consists of seven chapters: In the first chapter we present the mathematical model given in [2] wich represents the chemical

and biological behaviour of the wine fermentation process with the addition of nitrogen at certain instant of time. Also, we will do the simulation of the mathematical model and then compare it with the experimental data of certain chemical concentrations of the wine fermentation.

The main purpose of Chapter 2 is to build a mathematical model to obtain a better approximation (with respect to the mathematical model in Chapter 1) to the behaviour of the wine fermentation process. This construction is constituted by certain parameters that will be identified solving numerically step by step some optimization problems in python language.

The main purpose of Chapter 3 is to study the mathematical model of Chapter 2 but adding control variables. It is made in a tank like the batch reactor but the difference is that it is added concentrations of N, S, E with certain velocity Q_1, Q_2 and is removed concentrations of N, X, Tr, S, E with certain velocity Q. We are going to assume that the input rate is equal to the output rate ($Q_1 + Q_2 = Q$), that is to say the volume is constant because $\frac{dV}{dt} = Q - (Q_1 + Q_2)$. A tank with this assumption is called continuous stirred tank bioreactor.

Assuming that a MIMO linear system control problem without constraint is solved, that is to say, it is finded a input u^{wc} such that the output y^{wc} stabilizes in a some sense at a known desirable constant value y^*. So, the main purpose of Chapter 4 is to find a input $\tilde{u}$ of the same linear system control problem with constraint (the input $\tilde{u}$ is restricted) such that the output y is close to y^{wc} as much as possible at each instant of time. This was done in the article [4] and we are going to do that with more details and give the implicit solution of an example.

The main purpose of Chapter 5 is to find the input $\tilde{u}$ of a single-input single output (SISO) nonlinear system control problem with constraints such that the output y stabilizes at a known desirable constant value y^* and is to close to y^{wc} as possible at each instant of time. This will be solved by linearizing the SISO system in a linear system and finally we will apply the anti-windup technique to this linear system studied in the previous chapter. This was done in the article [5] and we are going to do that with more details and give the implicit solution of an example.

The main purpose of Chapter 6 is to find the input $\tilde{u}$ of a multi-input multi-output (MIMO) nonlinear system control problem with constraints such that the output y is to close to y^{wc} at each instant of time. The solution of this control problem is a generalization of the case SISO nonlinear system control problem with constraints. It was done in the article [6] and we are going to do that with more details and give the implicit solution of an example.

The main purpose of Chapter 7 is to control the wine fermentation with oenological condition represented for the mathematical model presented in Chapter 3, that is to say, we are going to add and remove the chemical concentrations of the continuous bioreactor with certain velocities Q_1 and Q_2 such that the sugar S and the rate production VCO_2 stabilizes at the desired values S^* and VCO_2^* respectively.

Chapter 1

Simulation of Malherbe model

1.1 Description of the Malherbe model

One of the most important steps of the wine production is the fermentation. The fermentation process has a difficult behaviour because of that is continuously studied to obtain a better mathematical model such that is approximated to the behaviour of the wine fermentation. The consequences of that are the minimization of the wine fermentation duration and the energy used in a cold tank or several tanks, it is realized by identification and automatic control tools by means of on-line monitoring of the fermentation process.

Yeast metabolism is very complex because of that the researchers try to make a mathematical model with specific conditions necessary for the fermentation control but unfortunately, most of the models are poorly adapted to enological conditions.

Generally, the Assimilable Nitrogen is not taken into account at all and the temperature effects are generally unsatisfactory, but these two variables are very important because their manipulations in experiments enable to control the reactions.

1.1.1 Problem

The aim is to make a mathematical model using the recent advances in physiological knowledge (principally the effect of nitrogen) where the rate production CO_2 is related to the effects of the main factors in wine-making conditions: nitrogen additions (which must not exceed the maximal authorized level) and temperature (which can vary within a predefined range).

1.1.2 Modeling

The main component to be measured in quantity is the CO_2 concentration or the rate production dCO_2/dt. In this model is measured the temperature in the tank and the addition of ammoniacal nitrogen during the fermentation. The addition has a dramatic impact on fermentation kinetics by increasing the fermentation rate, lowering the duration of the fermentation.

We present the dynamical model of the wine fermentation, with temperature and added ammoniacal nitrogen as the input variables and rate of CO_2 production as the output variable.

$$\begin{array}{rcl} \frac{dS}{dt} & = & -2.17X(t)\nu_{ST}[S(t),E(t),T]N_{ST}[N_{max}(t)-N(t),X(t),T] \\ \frac{dN}{dt} & = & -X(t)\nu_N[N(t),E(t),T] \\ \frac{dX}{dt} & = & k_1(T)X(t)\left[1-\frac{X(t)}{X_{max}(N_{init})}\right] \\ S(0) & = & S_{init} \\ N(0) & = & N_{init} \\ X(0) & = & X_{init} \end{array} \tag{1.1}$$

where $t \geq 0$; $X(t)$ yeast concentration is the cell population in the tank (cell/l); the maximum yeast concentration in a process with initial nitrogen concentration N_{ini} is denoted by $X_{max}(N_{init})$ (function depending of N_{init}); $S(t)$ is the glucose concentration in the tank (g/l); $E(t)$ is the ethanol concentration in the tank (g/l); $N_{max}(t) = N_{init}+N_{add}(t)$ where N_{add} is the amount of nitrogen added in the tank; $T(t)$ temperature in the tank (°C) and $k_1(T)$ is the growth rate of the population in the exponential growth phase.

The functions ν_{ST}, N_{ST} and ν_N are described in what follows. The concentrations of glucose and ethanol are deduced directly from the amount of CO_2 released. Assuming Gay-Lussac's law, we obtain the following equations:

$$\left\{ \begin{array}{rcl} S(t) & = & S(0)-2.17\mathrm{CO}_2(t) \quad \mathrm{CO}_2(0)=0 \\ E(t) & = & 0.464[S(0)-S(t)] \end{array} \right. \tag{1.2}$$

The yield coefficients (2.17 and 0.464) in the equation (1.2) are obtained by the work done in the paper [3].

1.1.3 Yeast activity

Yeast activity is implicitly described by four subsystems: glucose transport, glycolysis, nitrogen transport and synthesis of glucose transporters.

Glucose Transport

The function ν_{ST} describes the activity of a glucose transporter with ethanol inhibition, where the glucose enters in the membrane then by means of the glycolysis it is transformed in ethanol and CO_2 release.

$$\nu_{ST}(S,E,T) = \frac{k_2(T)S}{S+K_S+K_{SI}SE^{\alpha_S}} \tag{1.3}$$

where K_S, K_{SI} and α_S are constants, and k_2 is the rate fermentation depending of temperature.

Glycolysis

Glycolytic enzymes have a high level of activity and are not inhibited by ethanol, it goes out of the membrane to obtain ethanol and CO_2 release.

Process of Nitrogen Assimilation

The function ν_N is related to the nitrogen absorption by the yeast which is strongly inhibited by ethanol.

$$\nu_N(N, E, T) = \frac{k_3(T)N}{N + K_N + K_{NI}NE^{\alpha_N}} \tag{1.4}$$

where K_N, K_{NI} and α_N are constants, and k_3 is the rate fermentation depending of temperature.

Synthesis of Glucose Transporters

The functions $N_{ST}[N_{max}(t), N(t), X(t), T_{ucd}]$ represents the mean number of transporters in a yeast. In this process, the cell transforms a fraction of the absorbed nitrogen into proteins that will permit the glucose transport. This relationship implies that N_{ST} depends on the nitrogen assimilated by a single cell and environmental conditions.

1.2 Simulation of the Malherbe model

In this section, we will present the simulation of the dynamical system (1.1) using the following parameters identified by Malherbe and the initial conditions.

1.2.1 Parameters with temperature constant

$k_1(T) = 0.0287T - 0.3762$
$X_{max}(N_{init}) = 10^9(-649N_{init}^2 + 698N_{init} + 7)$
$k_2(T) = exp\left(\frac{-K_2}{T+273.15}\right)$ where $K_2 = 7000$
$K_S = 15 \quad K_{SI} = 0.012 \quad \alpha_S = 1.25$
$k_3 = 10^{-12} \quad K_N = 0.03 \quad K_{NI} = 0.035 \quad \alpha_N = 1.5$
$N_{ST}[N_{max(t)} - N(t), X(t), T] \times k_2(T) = \lambda_a \frac{N_i(t)}{X(t)} + \lambda_b T + \lambda_c \frac{N_i(t)}{X(t)} T + \lambda_d$
where $N_i = N_{init} + \lambda_0(N_{add})[N_{add}(t) - N(t)]$
$\lambda_0 = 3.2$ when the nitrogen addition is 0.063 g/l and $\lambda_0 = 0$ without nitrogen addition.
$\lambda_a = 335 \times 10^9 \quad \lambda_b = 0.061 \; \lambda_c = 3 \times 10^9 \quad \lambda_d = -1.$

1.2.2 Initial conditions

$S_{init} = 200.0$ g/l
$X_{init} = 9 \times 10^7$ cell/l.

1.2.3 Simulation to 24 °C

We present the simulation of Malherbe model using the parameters and initial conditions in the previous subsections.

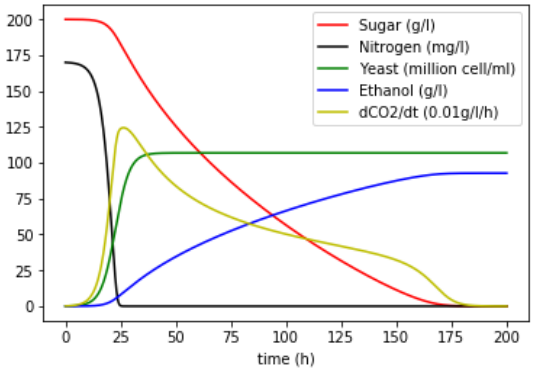


Figure 1.1: Simulation of a fermentation with $N_{init} = 0.17$ g/l.

Yeasts reproduce themselves because they have a logistic behaviour and as they grow fast, the nitrogen is consumed quickly. The sugar is consumed to produce ethanol and carbon dioxide. The rate production increases until 25h approximately and then it decreases because the sugar is concave from 0 to 25h and then it is convex.

1.3 Comparison of the Malherbe model and the experimental data

In this section, we will compare the approach of the simulation of Malherbe model with the experimental data for the rate production and the yeast.

1.3.1 Data of the rate production

The wine fermentation data are obtained through an experiment realized in a bath reactor, for more details read Chapter one of [1]. In this part, we present the rate production data of fourteen experiments to know its behaviour. There are several experiments where nitrogen is added at an instant of time and the rate production increases very fast.

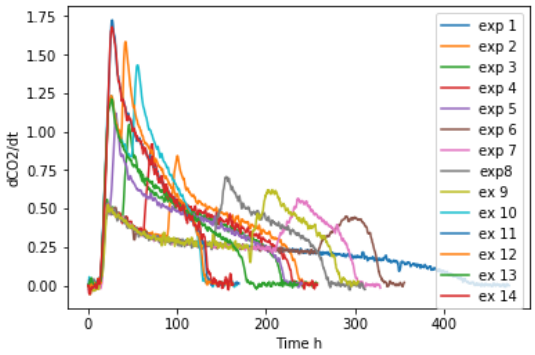


Figure 1.2: Data of the rate production ($d\mathrm{CO}_2/dt$).

We present the approach of the simulation of Malherbe model with the experimental data for fifteen experiments:

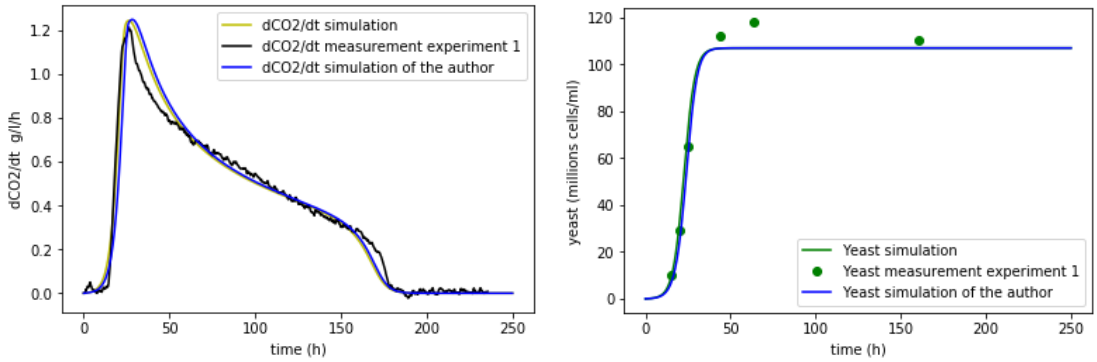


Figure 1.3: Rate production and yeast simulation of the first experiment with $N_{init} = 0.17$ g/l without nitrogen addition.

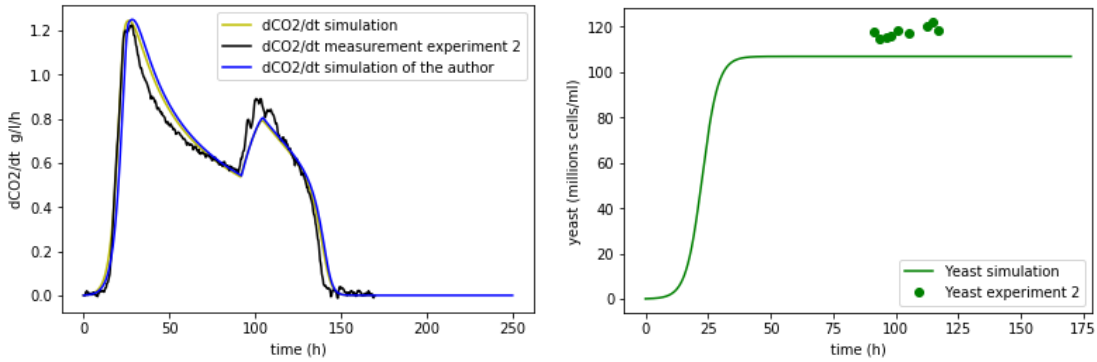


Figure 1.4: Rate production and yeast simulation of the second experiment with $N_{init} = 0.17$ g/l and $N_{add} = 0.063$ g/l at the instant $t = 91.47$ h.

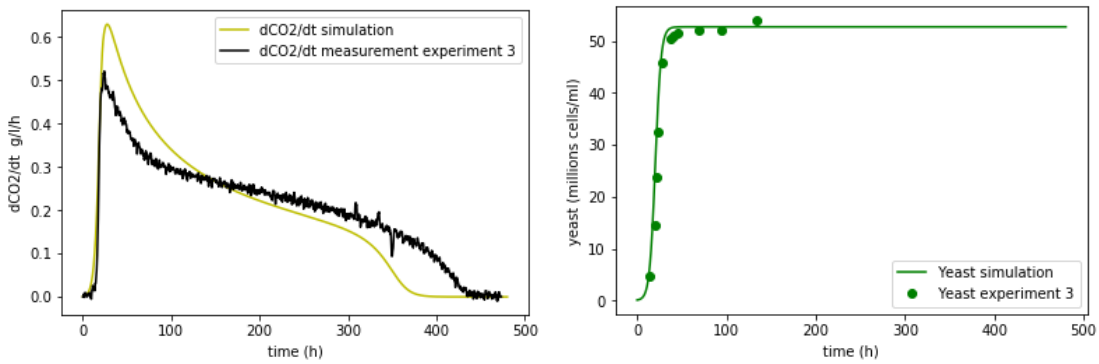


Figure 1.5: Rate production and yeast simulation of the third experiment with $N_{init} = 0.07$ g/l without nitrogen addition.

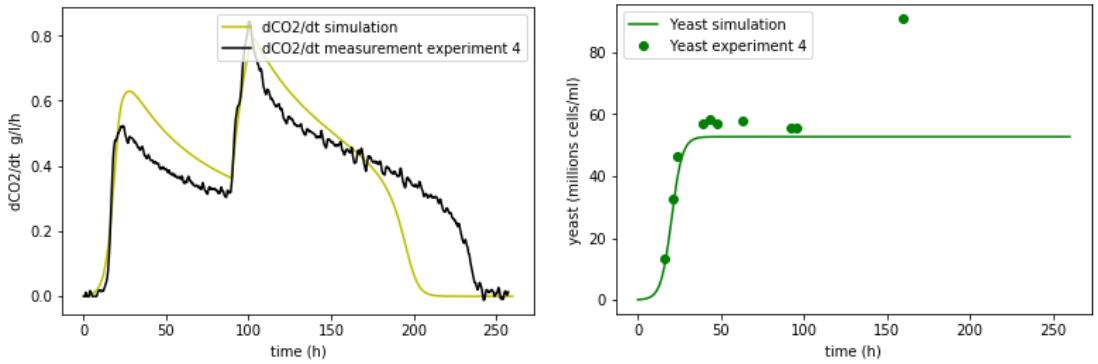


Figure 1.6: Rate production and yeast simulation of the fourth experiment with $N_{init} = 0.17$ g/l and $N_{add} = 0.063$ g/l at the instant $t = 89.53$ h.

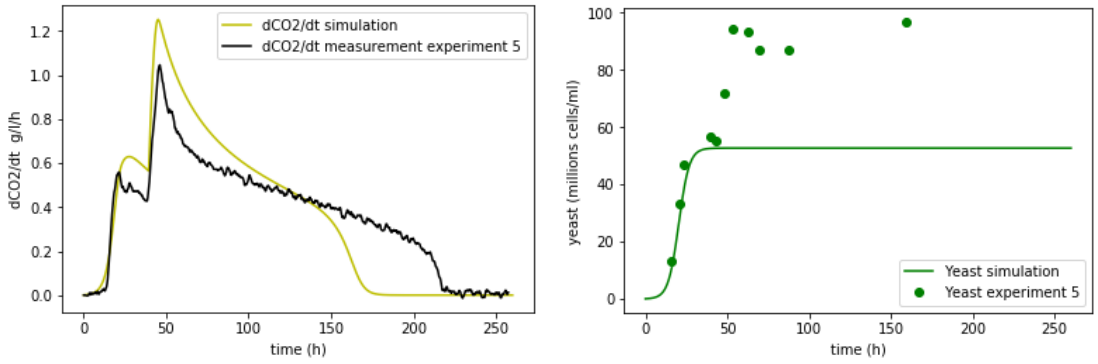


Figure 1.7: Rate production and yeast simulation of the fifth experiment with $N_{init} = 0.17$ g/l and $N_{add} = 0.063$ g/l at the instant $t = 39.4$ h.

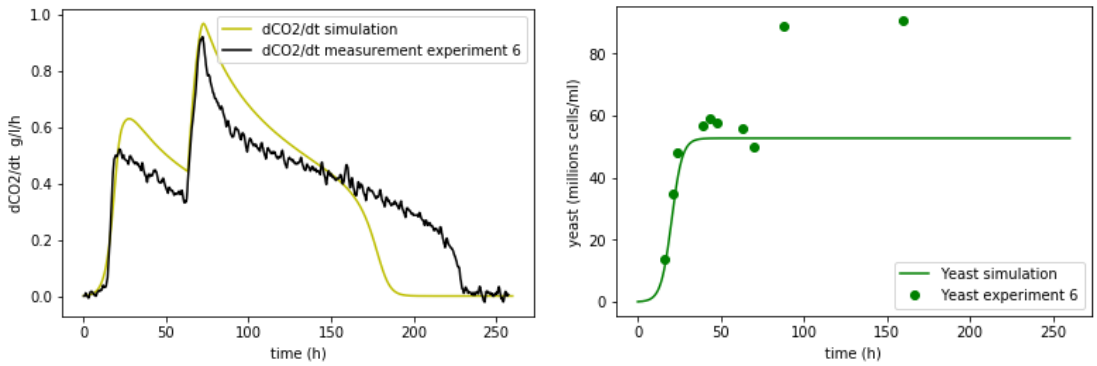


Figure 1.8: Rate production and yeast simulation of the sixth experiment with $N_{init} = 0.17$ g/l and $N_{add} = 0.063$ g/l at the instant $t = 63.11$ h.

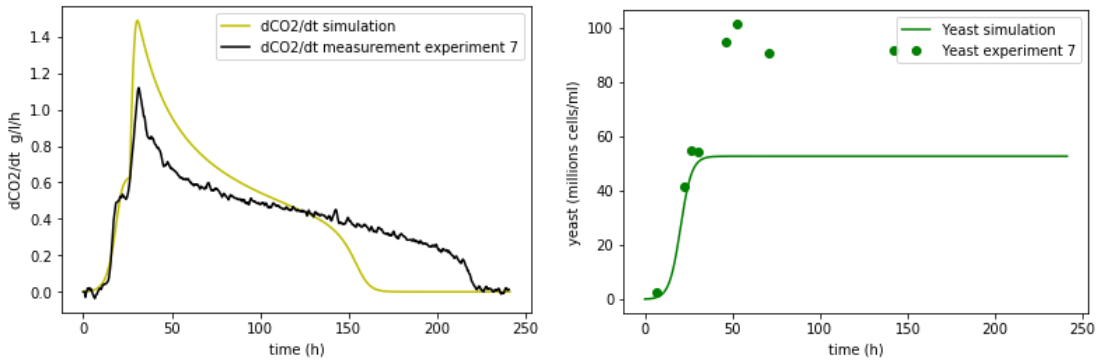


Figure 1.9: Rate production and yeast simulation of the seventh experiment with $N_{init} = 0.17$ g/l and $N_{add} = 0.063$ g/l at the instant $t = 26.3$ h.

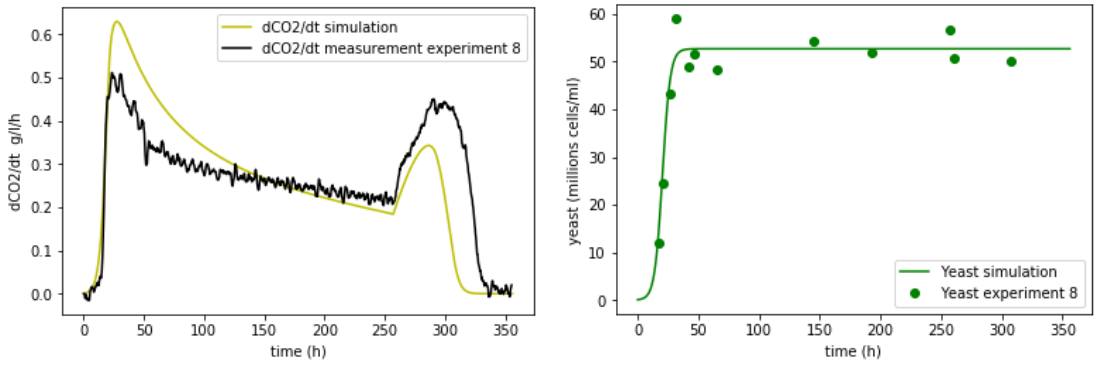


Figure 1.10: Rate production and yeast simulation of the eighth experiment with $N_{init} = 0.17$ g/l and $N_{add} = 0.063$ g/l at the instant $t = 257$ h.

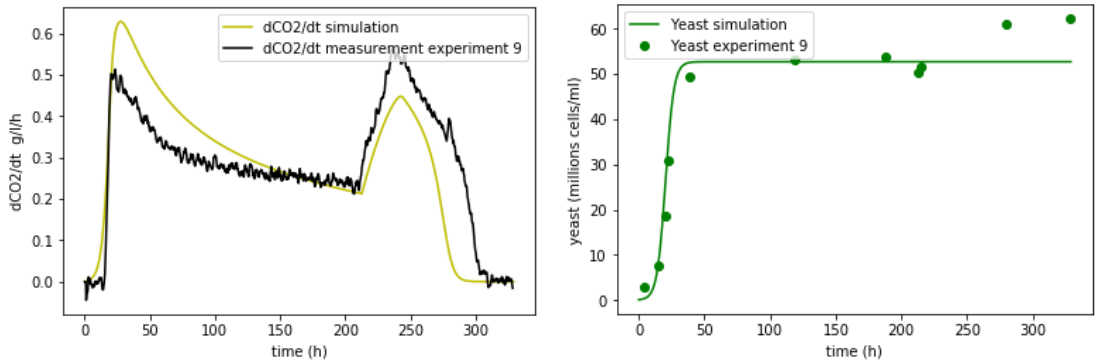


Figure 1.11: Rate production and yeast simulation of the nineth experiment with $N_{init} = 0.17$ g/l and $N_{add} = 0.063$ g/l at the instant $t = 212.63$ h.

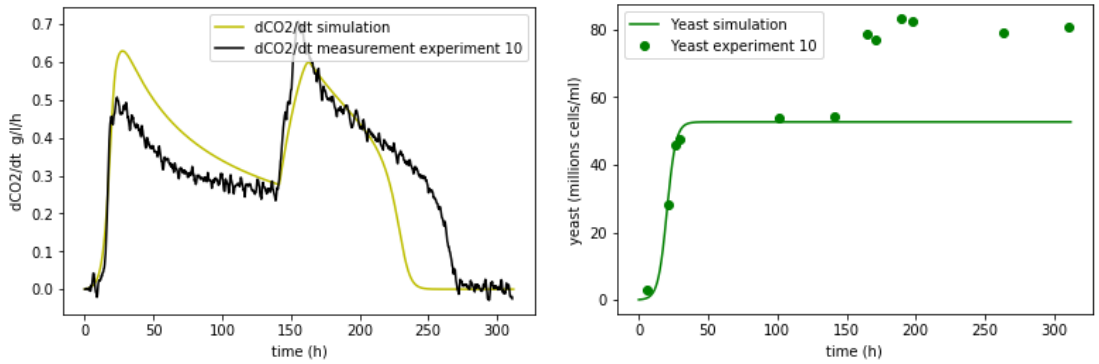


Figure 1.12: Rate production and yeast simulation of the tenth experiment with $N_{init} = 0.17$ g/l and $N_{add} = 0.063$ g/l at the instant $t = 141.51$ h.

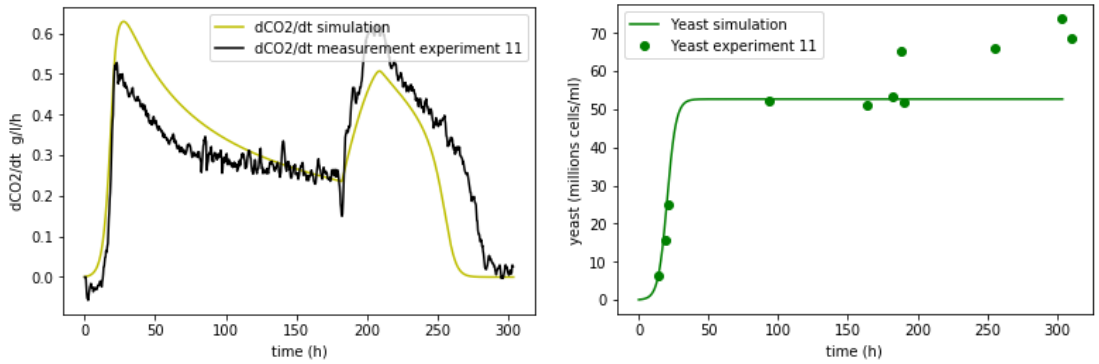


Figure 1.13: Rate production and yeast simulation of the eleventh experiment with $N_{init} = 0.17$ g/l and $N_{add} = 0.063$ g/l at the instant $t = 182.38$ h.

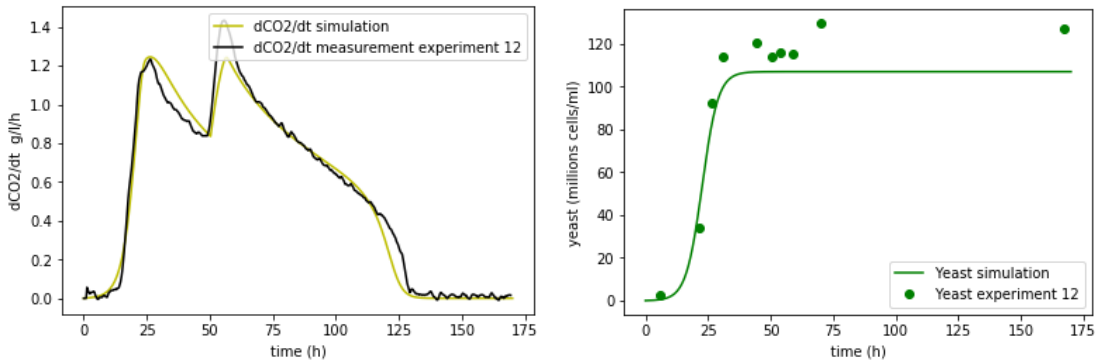


Figure 1.14: Rate production and yeast simulation of the twelfth experiment with $N_{init} = 0.17$ g/l and $N_{add} = 0.063$ g/l at the instant $t = 50.31$ h.

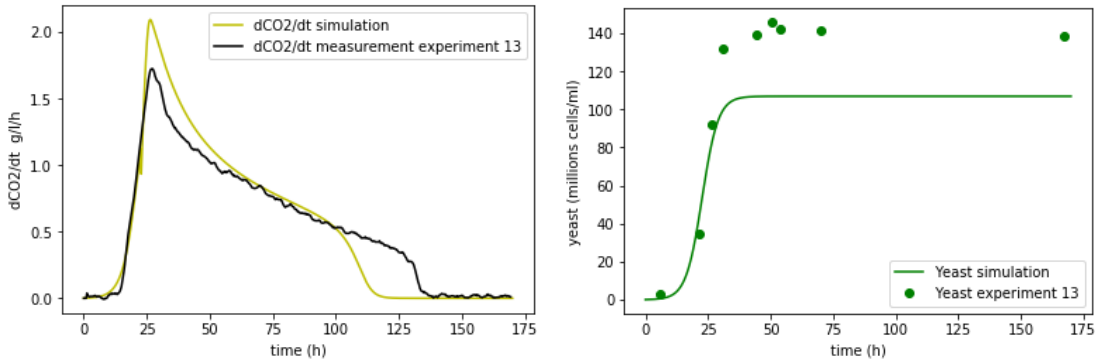


Figure 1.15: Rate production and yeast simulation of the thirteenth experiment with $N_{init} = 0.17$ g/l and $N_{add} = 0.063$ g/l at the instant $t = 22.81$ h.

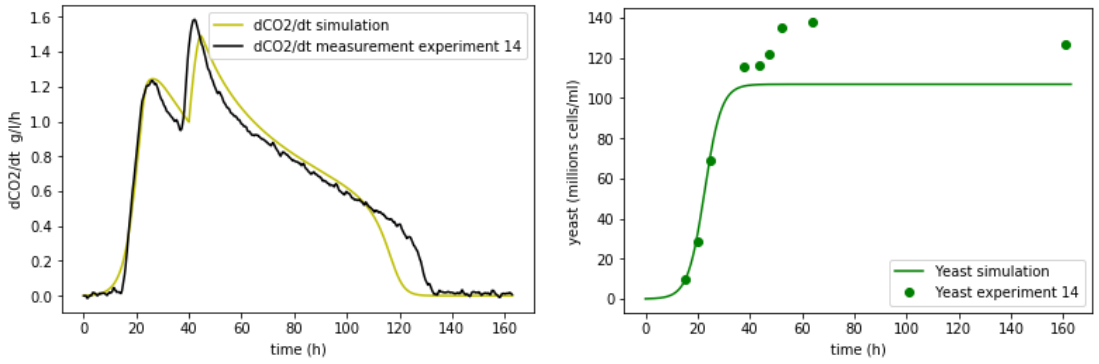


Figure 1.16: Rate production and yeast simulation of the fourteenth experiment with $N_{init} = 0.17$ g/l and $N_{add} = 0.063$ g/l at the instant $t = 39.9$ h.

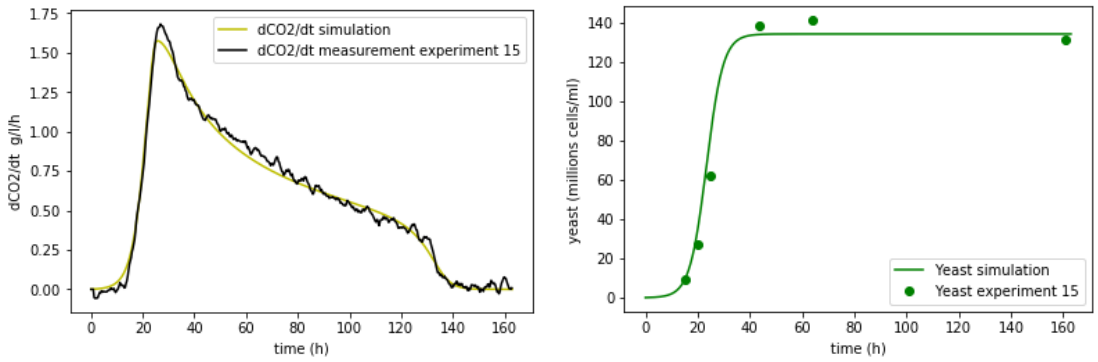


Figure 1.17: Rate production and yeast simulation of the fifteenth experiment with $N_{init} = 0.17$ g/l and $N_{add} = 0.063$ g/l at the instant $t = 0$ h.

In the experiments 1 and 2, the simulation of the rate production and yeast approximate very well to the experimental datas, but in the followings experiments they do not work well. The rate production does not work well in its maximum value and in the final part of time, and yeast does not work well when there is nitrogen addition.

1.4 Comments about the quality of the model

- There are some experiments where the model does not fit very well.

- The dynamic equation (1.1) of the yeast only depends on the yeast and has a logistic behaviour, because of that, yeast does not reproduce. We prove that mathematically, analyzing the equilibrium points:

$$-X\nu_{ST}N_{ST} = 0 \tag{1.5}$$
$$-X\nu_N = 0 \tag{1.6}$$
$$k_1[T]X[1 - \frac{X}{X_{max}(N_{init})}] = 0 \tag{1.7}$$

Biologically, we are interesed when $X \neq 0$. Then from equations (1.6) and (1.7), we have respectively $\nu_N = 0$ and $X = X_{max}(N_{init})$. Therefore the equilibrium points are $[S, N, X]_{eq} = [S, 0, X_{max}(N_{init})]$. Analyzing the stability, $\dot{x} = f(x)$ where

$x = [S, N, X]^T$, $f = [f_1, f_2, f_3]$,

$$
\begin{aligned}
f_1 &= -X\nu_{ST}N_{ST} \\
f_2 &= -X\nu_N \\
f_3 &= k_1[T]X[1 - \tfrac{X}{X_{max}(N_{init})}] \\
\frac{\partial f_1}{\partial S} &= -\frac{XK_2(T)K_S N_{ST}}{(S + K_S + K_{SI}SE^{\alpha_S})^2} < 0 \\
\frac{\partial f_2}{\partial S} &= 0 \\
\frac{\partial f_2}{\partial N} &= -\frac{Xk_3[T]K_N}{(N + K_N + K_{NI}NE^{alpha_N})^2} < 0 \\
\frac{\partial f_3}{\partial S} &= 0 \\
\frac{\partial f_3}{\partial N} &= 0 \\
\frac{\partial f_3}{\partial X} &= k_1[T][X_{max}(N_{init}) - \frac{1}{X_{max}(N_{init})}]
\end{aligned}
$$

$$
\frac{\partial f}{\partial x}(x_{eq}) = \begin{bmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial N} & \frac{\partial f_1}{\partial X} \\ 0 & \frac{\partial f_2}{\partial N} & \frac{\partial f_2}{\partial X} \\ 0 & 0 & -k_1[T] \end{bmatrix}
$$

The eigenvalues in the equilibrium point x_{eq} are $\frac{\partial f_1}{\partial S}$, $\frac{\partial f_2}{\partial N}$ and $-k_1[T]$ negatives. Therefore, the equilibrium point is stable. So that when there is not nitrogen addition, the Yeast will be constant igual to $X_{eq} = X_{max}(N_{init})$ and when we add nitrogen, the yeast will continue being igual $X_{max}(N_{init})$ because only depends of the N_{init}. Therefore when we add nitrogen, the yeast does not increase and does not represent a well fit.

Chapter 2

Improvement of Malherbe model

The purpose of this chapter is to improve the Malherbe model studied in Chapter 1. This improvement leads us to consider a new model which is made step by step where there are parameters that will be identified. The parameter identifications are obtained by solving an optimization problem that will be solved numerically using python language.

2.1 Description of the new model

In this section we present the mathematical model and the biological and chemical behaviour of the new model. In fact, the mentioned new model can be set as:

$$\begin{aligned} \frac{dX}{dt} &= \mu(N)Xf(S) \\ \frac{dN}{dt} &= -\mu(N)X \\ \frac{dS}{dt} &= -2.17X(t)\nu_{ST}Tr \\ \frac{dTr}{dt} &= c_1\mu(N)X[c_2 - f(S)] + g(E)Tr \end{aligned}$$

$$[X(0), N(0), S(0), Tr(0)] = [X_{init}, N_{init}, S_{init}, Tr_{init}]$$

where $X(t)$ (yeast concentration) is the cell population in the tank ($cell/l$); $N(t)$ and $S(t)$ are respectively the nitrogen and sugar concentration (g/l); and $Tr(t)$ (glucose transporter) is the number of sugar transporters in a cell. On the other hand, $\mu(N)$, $f(S)$ and $g(E)$ are defined as:

$$\begin{aligned} \mu(N): &= \frac{\mu_{max}N}{k_n + N} \\ f(S): &= aS^2 + bS + c \\ g(E): &= lE^2 + mE + n. \end{aligned} \tag{2.1}$$

where μ_{max}, k_n, a, b, c, l, m, and n are positive real numbers.

It is well know that the most important chemical reactions produced in the wine fermentation process are:

$$N \xrightarrow{X} X + Tr \qquad \text{Growth of X} \tag{2.2}$$

$$S \xrightarrow{X,Tr} E + CO_2 \qquad \text{Degradation of } S \text{ in ethanol} \tag{2.3}$$

So, by taking into account the Gay-Lussac's law of the chemical reaction (2.3), we get the following equations:

$$\begin{cases} S(t) &= S(0) - 2.17\mathrm{CO}_2(t) \quad \mathrm{CO}_2(0) = 0 \\ E(t) &= 0.464[S(0) - S(t)] \end{cases}$$

where the yield coefficients (2.17 and 0.464) in the system are obtained by mean of experiment results as studied in [3].

In (2.2), the nitrogen is consumed to produce yeast and glucose transporter in a yeast. The proportion $\frac{1}{2}f(S)$ of nitrogen rate is used by the yeast for its growth and the complement proportion $(1-\frac{1}{2}f(S))$ of nitrogen rate is used by the yeast for the synthesis of glucose transporter. The sugar is respectively absorbed and inhibited by the yeast and ethanol, this activity is described by ν_{ST} studied in Subsection 1.1.3, then the glucose transporter permits that the sugar enters with greater velocity in the yeast. When $g(E) < 0$, there is inhibition or degradation and when $g(E) > 0$, there is regeneration of transporter by the yeast to adapt to the environment.

2.2 Parameter identifications of the model

In this section, we describe a method to identify the parameters involving in the mathematical model proposed in Section 2.1. This method consists in minimizing a quadratic error of estimation.

2.2.1 Parameter identifications of $\mu(N)$ and $f(S)$

In this part, we are going to identify the parameters k_n, μ_{max}, x_{init}, a, b and c of the following dynamical system using two experimental data: the nitrogen data and the yeast data.

$$\begin{aligned} \frac{dN}{dt} &= -\mu(N)X \\ \frac{dX}{dt} &= \mu(N)Xf(S) \\ N(0) &= N_{init} \\ X(0) &= X_{init} \end{aligned}$$

where

$$\begin{aligned} \mu(N): &= \frac{\mu_{max}N}{k_n + N} \\ f(S): &= aS^2 + bS + c \end{aligned} \tag{2.4}$$

Method to identify the parameters

We use the traditional weighted least squares method to identify the parameters k_n, μ_{max}, x_{init}, a, b and c. The idea is to minimize the following quadratic error of estimation:

$$\left.\begin{array}{lll} \min\limits_{(k_n,\mu_{max},X_{init},a,b,c)} & w_1||Ndata - Nsim||_2^2 + w_2||Xdata - Xsim||_2^2 & \\ & \dfrac{dN}{dt} = -\mu(N)X & \\ \text{s.t.} & \dfrac{dX}{dt} = \mu(N)Xf(S) & \\ & N(0) = N_{init} & \\ & X(0) = X_{init} & \end{array}\right\} \quad (2.5)$$

where w_1 and w_2 are the weights (fixed positive numbers), and

$$\begin{aligned} Ndata &:= [(Ndata)_1, \cdots, (Ndata)_n] \\ Nsim &:= [N(t_1), \cdots, N(t_n)] \\ Xdata &:= [(Xdata)_1, \cdots, (Xdata)_m] \\ Xsim &:= [X(t_1), \cdots, X(t_m)] \end{aligned}$$

where $(Ndata)_i$ and $(Xdata)_i$ are respectively the data of nitrogen and yeast at time t_i.

So for instance, using $Ndata$ and $Xdata$ vectors borrowed from [2], $w_1 = 0.01$, $w_2 = 10$, $n = 9$ and $m = 12$, the numerical result (using Python language) of problem (2.5) is:

$$\begin{pmatrix} X_{init} \\ k_n \\ \mu_{max} \\ a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 3 \\ 0.35620067289411805 \\ 0.00074179295965841698 \\ 0.015807664957978633 \\ 9.3168038110079628e-07 \\ 212.34153088786363 \end{pmatrix}. \quad (2.6)$$

Simulation results for yeast and nitrogen experiments

Each one of the next two graphic describes the relationship between the data collections and the simulation approach. One of the graphics corresponds to the yeast and the other one to the nitrogen.

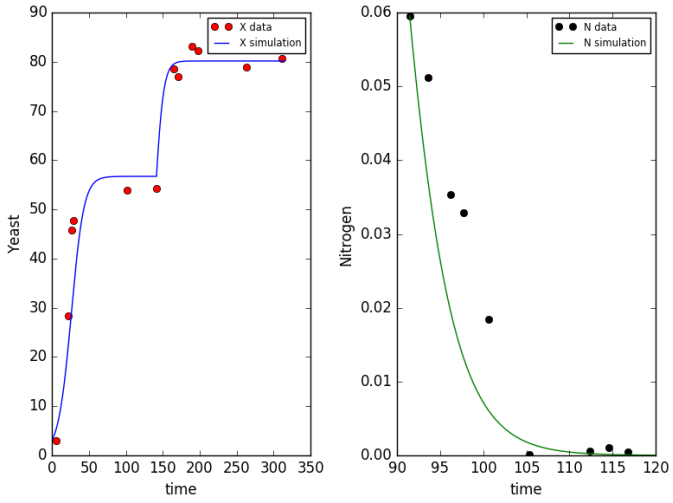


Figure 2.1: Identification of $\mu(N)$ and $f(S)$.

2.2.2 Paremeter identifications of $f(S)$

Similar to the previous analysis we use the following quadratic optimization problem in order to identify the parameters a, b and c from $f(S)$ using the yeast data.

$$\left.\begin{array}{lll} \min\limits_{(a,b,c)} & \sum\limits_{k=1}^{r} ||Xdata_k - Xsim_k||_2^2 & \\ & \dfrac{dX_k}{dt} = \mu(N_k)X_k f(S_k) & \\ \text{s.t.} & \dfrac{dN_k}{dt} = -\mu(N_k)X_k & \\ & X_k(0) = X_{k_{init}} & \\ & N_k(0) = N_{k_{init}} & \end{array}\right\} \tag{2.7}$$

where for $k = 1, \cdots, r$, the functions $\mu(N_k)$ and $f(S_k)$ are defined in (2.1), and

$$\begin{aligned} Xdata_k &:= [(Xdata_k)_1, \cdots, (Xdata_k)_{m_k}]; \\ Xsim_k &:= [X_k(t_1), \cdots, X_k(t_{m_k})]. \end{aligned}$$

Here $(Xdata_k)_i$ is the data of yeast at time t_i of the kth experiment.

So for instance, by considering $r = 9$, $m_1 = m_3 = m_7 = m_9 = 12$; $m_2 = m_4 = 10$; $m_5 = 8$ and $m_6 = m_8 = 12$, as studied in [2] and also by taking $Xdata_k$ from this reference, the numerical solution of (2.7) is:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2.01142324e-02 \\ 7.53592364e-07 \\ 5.10588530e+01 \end{pmatrix}.$$

Here, the parameters μ_{max} and k_n in $\mu(N_k)$ are the same of (2.6).

Simulation results for the yeast experiments

The following graphic describes the approach of the yeast simulation with yeast data of nine experiments each identified with different color.

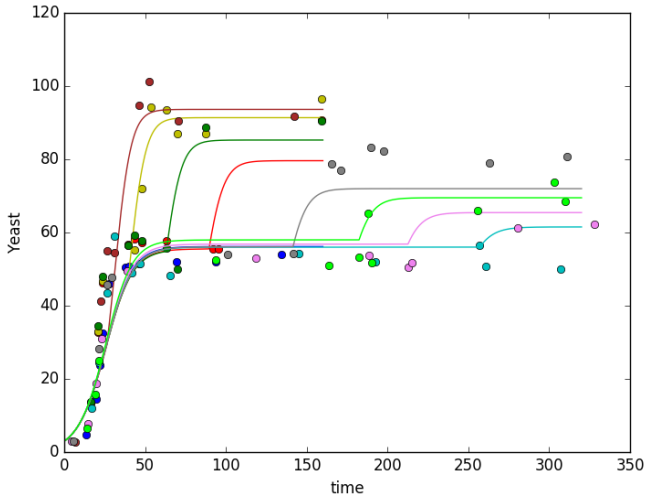


Figure 2.2: Identification of $f(S)$ using yeast data of all the experiments.

2.2.3 Parameter identifications of Glucose Transporter

Similar to the previous subsection, we will identify the paremeters c_1, c_2, l, m, and n of the following dynamical system using the glucose transporter data.

$$\begin{aligned}
\frac{dTr}{dt} &= c_1\mu(N)X[c_2 - f(S)] + g(E)Tr \\
\frac{dX}{dt} &= \mu(N)Xf(S) \\
\frac{dN}{dt} &= -\mu(N)X \\
Tr(0) &= Tr_{init} \\
X(0) &= X_{init} \\
N(0) &= N_{init}
\end{aligned}$$

where

$$g(E) := lE^2 + mE + n.$$

Method to identify the parameters

Similar to the previous subsection, we identify the parameters c_1, c_2, l, m, and n using the glucose transporter data solving the following quadratic optimization problem.

$$
\left.
\begin{array}{lll}
\min\limits_{(c_1,c_2 l,m,n)} & \displaystyle\sum_{k=1}^{r} ||Trdata_k - Trsim_k||_2^2 & \\
& \dfrac{dTr_k}{dt} &= c_1\mu(N_k)X_k[c_2 - f(S_k)] + g(E_k)Tr_k \\
& \dfrac{dX_k}{dt} &= \mu(N_k)X_k f(S_k) \\
\text{s.t.} & \dfrac{dN_k}{dt} &= -\mu(N_k)X_k \\
& Tr_k(0) &= Tr_{k_{init}} \\
& X_k(0) &= X_{k_{init}} \\
& N_k(0) &= N_{k_{init}}
\end{array}
\right\} \tag{2.8}
$$

where

$$
\begin{aligned}
Trdata_k &:= [(Trdata_k)_1, \cdots, (Trdata_k)_{m_k}] \\
Trsim_k &:= [Tr_k(t_1), \cdots, Tr_k(t_{m_k})]
\end{aligned}
$$

where $(Trdata)_{ki}$ is the data of glucose transporter at time t_i of the kth experiment.

So for instance, using $Trdata$ borrowed from [2], and considering, $r = 9$ and $m_k = 1422$, the numerical solution of problem (2.8) is:

$$
\begin{pmatrix} c_1 \\ c_2 \\ l \\ m \\ n \end{pmatrix} = \begin{pmatrix} 2.99621981e-02 \\ 1.44460425e+03 \\ -1.31367315e-06 \\ 1.50119898e-04 \\ -3.06341075e-03 \end{pmatrix}.
$$

Simulation results of glucose transporter experiments

The following graphic describes the approach of the glucose transporter with the glucose tranporter data of nine experiments.

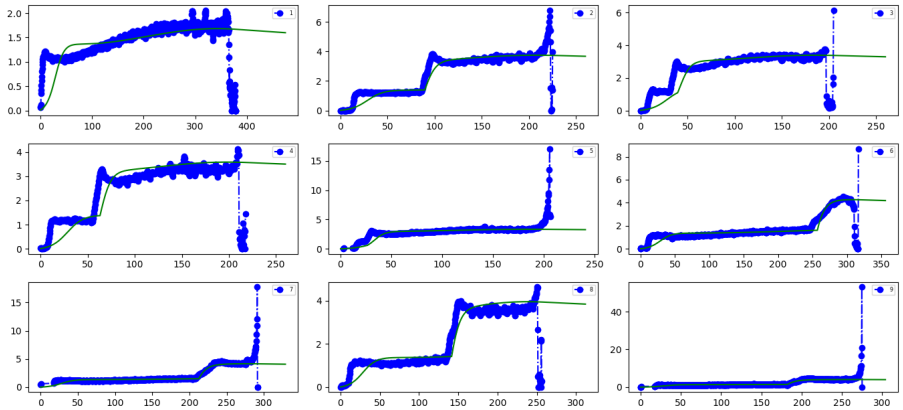


Figure 2.3: Identification of Glucose Transporter using yeast data of nine experiments.

2.3 Simulation of the new mathematical model

In this section, we present the simulation of the mathematical model presented in the Section 2.1 to compare with the experimental data.

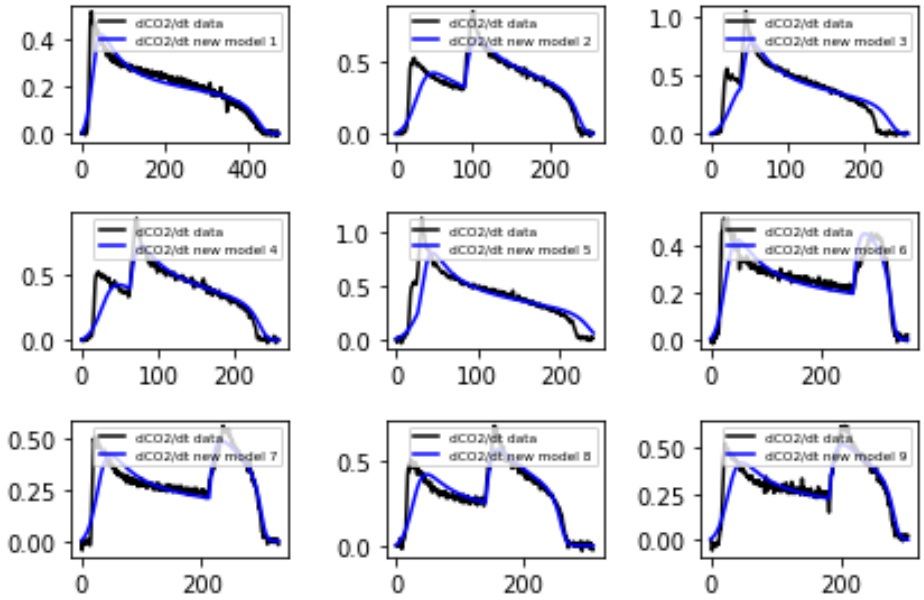


Figure 2.4: Comparison between the rate production simulation and the rate production data.

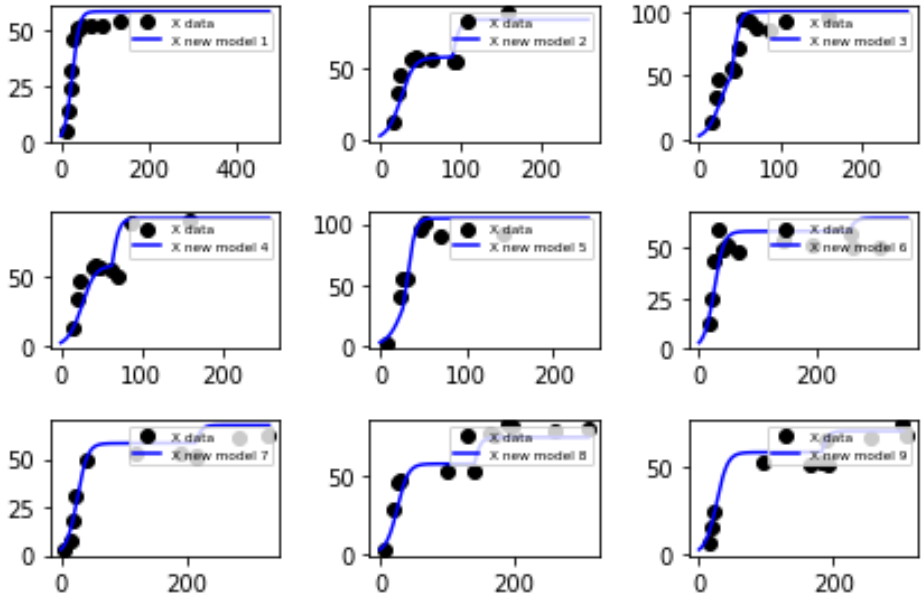


Figure 2.5: Comparison between the yeast simulation and the yeast data.

2.3.1 Comparing the two mathematical models together the experimental data

We present here the simulation results obtained from the mathematical model presented in the Section 2.1 and the malherbe model described in equation (1.1), and we compare they together with the experimental data.

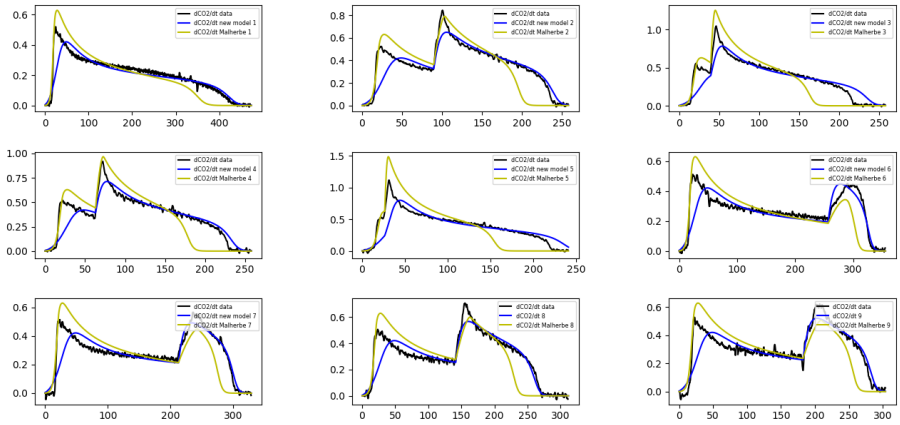


Figure 2.6: Comparison between the rate production simulation of the new model, rate production simulation of the Malherbe model and the rate production data.

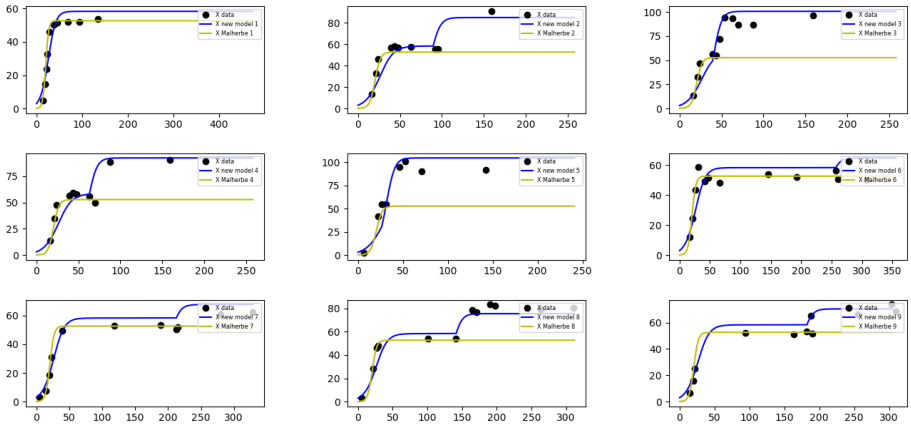


Figure 2.7: Comparison between the yeast simulation of the new model, yeast simulation of the Malherbe model and the yeast data.

Chapter 3

Continuous stirred tank bioreactor for the new mathematical model

In Chapter 2 we made a mathematical model to study the wine fermentation process with oenological conditions (nitrogen addition), the experiments are made in a batch reactor where there are concentrations in the tank. The main purpose of this chapter is to study this mathematical model but adding control variables. It is made in a tank like the batch reactor but the difference is that it is added concentrations of N, S, E with certain velocity Q_1, Q_2 and is removed concentrations of N, X, Tr, S, E with certain velocity Q. We are going to assume that the input rate is equal to the output rate ($Q_1 + Q_2 = Q$), that is to say the volume is constant because $\frac{dV}{dt} = Q - (Q_1 + Q_2)$. A tank with this assumption is called continuous stirred tank bioreactor.

3.1 Mass balance

In this section, we use the mass balance law to the chemical concentrations involving in the dynamical system of the Section 2.1. Therefore, the dynamical behaviour of the nitrogen is equal to $-\mu(N)XV$ which is the rate of nitrogen comsuption in the tank; $N_{in}Q_1 + N_{add}Q_2$ is the nitrogen rate that come into the tank with different dilution rate Q_1 and Q_2; and finally $-NQ$ is the nitrogen rate leaving the tank. The dynamical behaviour of the other concentrations have the same structure.

$$\left.\begin{array}{rcl} \frac{d(NV)}{dt} &=& -\mu(N)XV + N_{in}Q_1 + N_{add}Q_2 - NQ \\ \frac{d(XV)}{dt} &=& \mu(N)f(S)XV - XQ \\ \frac{d(TrV)}{dt} &=& [c_1\mu(N)X(c_2 - f(S)) + g(E)Tr]V - TrQ \\ \frac{d(SV)}{dt} &=& -\nu(E,S)TrV + S_{in}Q_1 - SQ \\ \frac{d(EV)}{dt} &=& 0.464\nu(E,S)TrV + E_{in}Q_1 - EQ \end{array}\right\} \tag{3.1}$$

Using the dynamical system of (3.1) and the assumption of constant volumen of the tank, we obtain the mathematical model with control variables as follows:

$$\left.\begin{array}{rcl} \frac{dN}{dt} &=& -\mu(N)X + (N_{in} - N)\frac{Q_1}{V} + (N_{add} - N)\frac{Q_2}{V} \\ \frac{dX}{dt} &=& \mu(N)f(S)X - X\frac{Q_1}{V} - X\frac{Q_2}{V} \\ \frac{dTr}{dt} &=& c_1\mu(N)X(c_2 - f(S)) + g(E)Tr - Tr\frac{Q_1}{V} - Tr\frac{Q_2}{V} \\ \frac{dS}{dt} &=& -\nu(E,S)Tr + (S_{in} - S)\frac{Q_1}{V} - S\frac{Q_2}{V} \\ \frac{dE}{dt} &=& 0.464\nu(E,S)Tr + (E_{in} - E)\frac{Q_1}{V} - E\frac{Q_2}{V} \end{array}\right\} \quad (3.2)$$

3.2 State - space representation of the wine fermentation with enological condition

The mathematical model described in (3.2) can be represented by a state - space representation as follows:

$$\left.\begin{array}{rcl} \frac{d\xi}{dt} &=& f(\xi) + g(\xi)u \\ y &=& h(\xi) \\ Q_1^{min} \leq u_1 \leq & & Q_1^{max} \\ Q_2^{min} \leq u_2 \leq & & Q_2^{max} \end{array}\right\} \quad (3.3)$$

where

$$\begin{aligned} \xi &:= [N, X, Tr, S, E]^t \\ f(\xi) &:= \begin{bmatrix} -\mu(N)X \\ \mu(N)f(S)X \\ c_1\mu(N)X(c_2 - f(S)) + g(E)Tr \\ 0.464\nu(E,S)Tr \end{bmatrix} \\ g(\xi) &:= \frac{1}{V}\begin{bmatrix} N_{in} - N & N_{add} - N \\ -X & -X \\ -Tr & -Tr \\ S_{in} - S & -S \\ E_{in} - E & -E \end{bmatrix} \\ u &:= [u_1, u_2]^t = [Q_1, Q_2]^t \\ VCO_2 &:= \tfrac{1}{2.17}\nu(E,S)Tr \\ h(\xi) &:= [S, VCO_2]^t \end{aligned}$$

Chapter 4

Anti-windup for Internal Model Control

Assuming that a MIMO linear system control problem without constraint is solved, that is to say, it is finded a input u^{wc} such that the output y^{wc} reaches and stabilizes at a known desirable constant value y^*. The main purpose of this chapter is to find a input $\tilde{u}$ of the same linear system control problem with constraint (the input $\tilde{u}$ is restricted) such that the output y is close to y^{wc} as possible at each instant of time. This was done in the article [4] and we are going to do that with more details and give the implicit solution of an example.

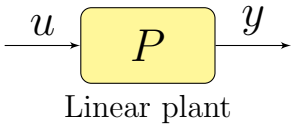
4.1 Introduction

Definición 4.1. *Let f be a real-valued, locally integrable function defined on the positive real numbers. Then*

$$F(s) = [\mathcal{L}(f)](s) := \int_0^\infty f(t)e^{-st}\,dt, \quad f = \mathcal{L}^{-1}[F]$$

denote, as usual, the direct and inverse Laplace transforms.

The scheme in Figure 4.1 is called open loop where u enters to the system P obtaining a response y, $u(t) := [u_1(t), ..., u_n(t)]^T \in \mathbb{R}^n$ and $y(t) := [y_1(t), ..., y_n(t)]^T \in \mathbb{R}^n$ are the input and the output respectively.



Linear plant

Figure 4.1: Linear system with input u and output y.

The elements of $P(s) \in \mathbb{R}^{n\times n}$ are quotient of polynomials and mathematically, the scheme of Figure 4.1 is the following

$$Y(s) = P(s)U(s) \tag{4.1}$$

where

$$Y(s) = \mathcal{L}\{y(t)\} := [Y_1(s), ..., Y_n(s)]^T \in \mathbb{R}^n \qquad Y_i(s) = \mathcal{L}\{y_i(t)\}$$

and

$$U(s) = \mathcal{L}\{u(t)\} := [U_1(s), ..., U_n(s)]^T \in \mathbb{R}^n \qquad U_i(s) = \mathcal{L}\{u_i(t)\}$$

are respectively the Laplace transforms of the output y and the input u. $P(s) = [P_{ij}(s)]$ is called transfer function.

Remark. *The equation (4.1) or the scheme of Figure 4.1 is equivalent to a system of linear ordinary differential equations.*

Definición 4.2. *A strictly proper transfer function is a transfer function where the degree of the numerator is less than the degree of the denominator. A proper transfer function is a transfer function where the degree of the numerator is less or equal to the degree of the denominator. A biproper transfer function is a transfer function where the degree of the numerator is equal to the degree of the denominator.*

We define the matrix $p(t) := [p_{ij}(t)] \in \mathbb{R}^{n\times n}$ whose $ij-$element $p_{ij}(t) = \mathcal{L}^{-1}[P_{ij}(s)]$ is the inverse Laplace transform of P_{ij}.

Definición 4.3 (The convolution transform). *The convolution transform of p and u, denoted by $p * u$, is defined on all $\mathbb{R}$ as*

$$(p * u)(t) := \int_0^t p(t-\tau)u(\tau)d\tau \tag{4.2}$$

We are going to prove that the inverse Laplace transform of the product of two Laplace transforms in (4.1) is equal to the convolution transform of the two inverse Laplace transforms.

From equation (4.1), we have the following:

$$\begin{aligned}
Y_i(s) &= \sum_{j=1}^{n} P_{ij}(s)U_j(s) \\
y_i(t) &= \sum_{j=1}^{n}(p_{ij} * u_j)(t) = \sum_{j=1}^{n}\int_0^t p_{ij}(t-\tau)u_j(\tau)d\tau \\
y_i(t) &= \int_0^t \sum_{j=1}^{n} p_{ij}(t-\tau)u_j(\tau)d\tau \\
y(t) &= \int_0^t v^t(\tau)d\tau
\end{aligned}$$

where

$$v^t(\tau) := \begin{bmatrix} \sum_{j=1}^{n} p_{1j}(t-\tau)u_j(\tau) \\ \vdots \\ \sum_{j=1}^{n} p_{nj}(t-\tau)u_j(\tau) \end{bmatrix}$$

From equation (4.2), we have that

$$y(t) = \int_0^t p(t-\tau)u(\tau)d\tau$$

$$\boxed{y(t) = (p * u)(t)}$$

4.2 Problem formulation

For $u \in \mathbb{R}^n$, $sat(u) := [sat(u_1), ..., u_{(n)}]^T$ where

$$sat(u_i) = \begin{cases} u_i^{max} & u_i > u_i^{max} \\ u_i & u_i^{min} \leq u_i \leq u_i^{max} \\ u_i^{min} & u_i < u_i^{min} \end{cases}$$

We assume that the MIMO linear system without constraint,

$$\left.\begin{array}{rcl} \dot{x} & = & Ax(t) + Bu(t) \\ y(t) & = & Cx(t) \end{array}\right\} \tag{4.3}$$

where A, B and C are given $n \times n$ real matrices with B and C invertible,
is solved. Therefore, the close loop scheme

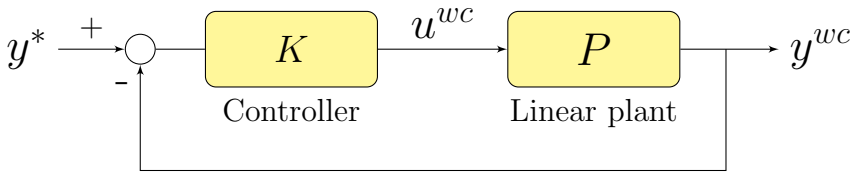


Figure 4.2: Closed loop of a linear system without constraints.

is the solution of the linear system (4.3). Where u^{wc} and y^{wc} are respectively the input and output.

From Figure 4.2, we have the following equations:

$$\begin{array}{rcl} Y^{wc} & = & PU^{wc} \\ U^{wc} & = & K(Y^* - Y^{wc}) \end{array} \tag{4.4}$$

where Y^{wc}, U^{wc} and Y^{wc} are respectively the Laplace transforms of y^{wc}, u^{wc} and y^*.

From equation (4.4),

$$Y^{wc} = P(I + KP)^{-1}KY^*$$

Let $F \in \mathbb{R}^{n\times n}$ be the Laplace transform of a function f. Therefore, using the previous equation

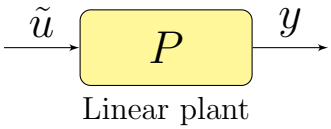
$$(f * y^{wc})(t) = \mathcal{L}^{-1}[FP(I + KP)^{-1}Ky^*](t)$$

The main purpose of this Chapter is to solve the MIMO linear system with constraint

$$\left.\begin{array}{lcl} \dot{x} & = & Ax(t) + Bu(t) \\ y(t) & = & Cx(t) \\ u^{min} & \leq u(t) \leq & u^{max} \end{array}\right\} \tag{4.5}$$

or equivalent to

$$\left.\begin{array}{lcl} \dot{x} & = & Ax(t) + B\tilde{u}(t) \\ y(t) & = & Cx(t) \\ \tilde{u}(t) & := & sat(u(t)) \end{array}\right\} \tag{4.6}$$



Linear plant

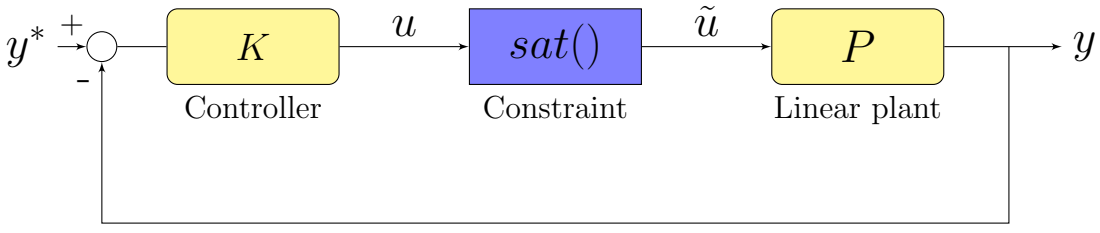
Figure 4.3: Linear system with constraint.

such that y is close to y^{wc} as possible at each time t, that is to say, mathematically we want to solve the following optimization problem at each time t,

$$\min_{\tilde{u}} ||(f * y_{wc})(t) - (f * y)(t)||_1 = \min_{\tilde{u}} ||\mathcal{L}^{-1}[FP(I + KP)^{-1}Ky^*](t) - (\mathcal{L}^{-1}[FP] * \tilde{u})(t)||_1, \tag{4.7}$$

where f is a diagonal filter such that FP is biproper.

In general the IMC scheme in Figure 4.4 solves (4.6) but does not solve the optimization problem (4.7), because of that we are going to present in the next section a close loop scheme satisfying certain conditions to resolve the problems (4.6) and (4.7).



Controller Constraint Linear plant

Figure 4.4: Conventional linear IMC scheme.

4.3 Anti-windup design

In this section we are going to present a close loop scheme satisfying certain conditions to solve the problems (4.6) and (4.7).

Definición 4.4 (Stability and minimum-phase). *A linear system is stable when all poles (roots of the denominator) of the elements of its transfer function have their negative real part. A linear system is minimum-phase when all zeros (roots of the numerator) of the elements of its transfer function have their negative real part.*

The following Lemma is proved in the article [4].

Lemma. *The following close loop*

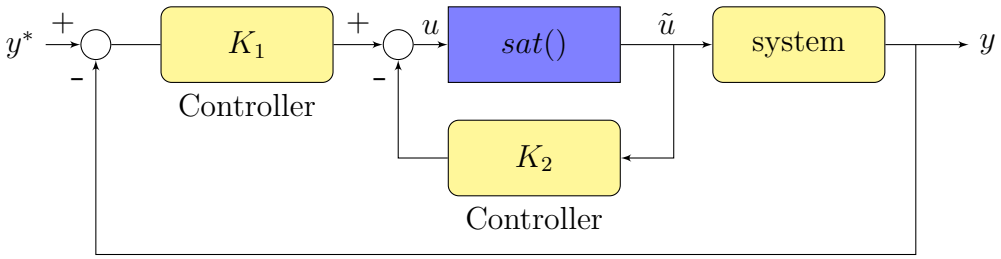


Figure 4.5: Anti-windup linear IMC scheme.

with:
$K_1 := FP(I + KP)^{-1}K$ *and* $K_2 := FP - I - K_1P$,
and the following assumptions:

1) $(I + KP)^{-1}K$ *is biproprer, stable and minimum-phase,*

2) $FP_{|s=\infty}$ *is a diagonal nonsingular matrix with finite elements,*

3) K_1 *is stable and minimum-phase,*

4) $K_1P + K_2$ *is strictly proper,*

solve (4.6) and the optimization problem (4.7).

4.4 Example

In this section, we are going to present a example and will show the perfomance of the Anti-windp method. We consider the following linear system,

$$\begin{array}{lll} \dot{y} & = & -0.01y + 0.02u \\ y(0) & = & 0 \\ -1 & \leq u \leq & 1 \end{array}$$

In this example, the setpoint y^* is equal to 1.

Then, the linear plant is the following:

$$P(s) = \frac{2}{100s+1}.$$

The IMC controller designed is

$$K = \frac{100s+1}{40s}.$$

Case 1. Chossing $f(s) = 2.5(20s+1)$ gives

$$\begin{aligned} K_1 &= 2.5 \\ K_2 &= \frac{-1}{100s+1} \end{aligned}$$

Case 2. Chossing $f(s) = 50(s+1)$ gives

$$\begin{aligned} K_1 &= \frac{50(s+1)}{20s+1} \\ K_2 &= \frac{18802-1}{(100s+1)(20s+1)} \end{aligned}$$

4.4.1 Close loop

a) Unconstrain system is obtained by the scheme in Figure 4.2:

$$\begin{aligned} Y &= PU \\ U &= K(Y^* - Y) \end{aligned}$$

The mathematical equations are the following:

$$\begin{aligned} \dot{y} &= -0.01y + 0.02u \\ u &= 2.5(y^* - y) + z \end{aligned}$$

where

$$\begin{aligned} \dot{z} &= 0.025(y^* - y) \\ [y(0), z(0)] &= [0, 0] \end{aligned}$$

b) IMC system is obtained by the scheme in Figure 4.4:

$$\begin{aligned} Y &= P\tilde{U} \\ \tilde{u} &= sat(u) \\ U &= K(Y^* - Y) \end{aligned}$$

The mathematical equations are the following:

$$\begin{aligned} \dot{y} &= -0.01y + 0.02\tilde{u} \\ \tilde{u} &= sat(u) \\ u &= 2.5(y^* - y) + z \end{aligned}$$

where

$$\begin{aligned} \dot{z} &= 0.025(y^* - y) \\ [y(0), z(0)] &= [0, 0] \end{aligned}$$

c) Anti-windup IMC case 1 is obtained by the scheme in Figure 4.5:

$$\begin{aligned} Y &= P\tilde{U} \\ \tilde{u} &= sat(u) \\ U &= K_1(Y^* - Y) - K_2\tilde{U} \end{aligned}$$

The mathematical equations are the following:

$$\begin{aligned} \dot{y} &= -0.01y + 0.02\tilde{u} \\ \tilde{u} &= sat(u) \\ u &= 2.5(y^* - y) + z \end{aligned}$$

where

$$\begin{aligned} \dot{z} &= 0.01(-z + \tilde{u}) \\ [y(0), z(0)] &= [0, 0] \end{aligned}$$

d) Anti-windup IMC case 2 is obtained by the scheme in Figure 4.5 :

The mathematical equations are the following:

$$\begin{aligned} \dot{y} &= -0.01y + 0.02\tilde{u} \\ \tilde{u} &= sat(u) \\ u &= 2.5(y^* - y) + 47.5z_1 + 24.75z_2 - 23.75z_3 \end{aligned}$$

where

$$\begin{aligned} \dot{z_1} &= 0.05(-z_1 + y^* - y) \\ \dot{z_2} &= 0.01(-z_2 + \tilde{u}) \\ \dot{z_3} &= 0.05(-z_3 + \tilde{u}) \\ [y(0), z_1(0), z_2(0), z_3(0)] &= [0, 0, 0, 0] \end{aligned}$$

4.4.2 Simulation results

We present the plots, imput and output versus time, for three cases: system without and with constraint solved by the IMC scheme, and finally the Anti-windup IMC scheme.

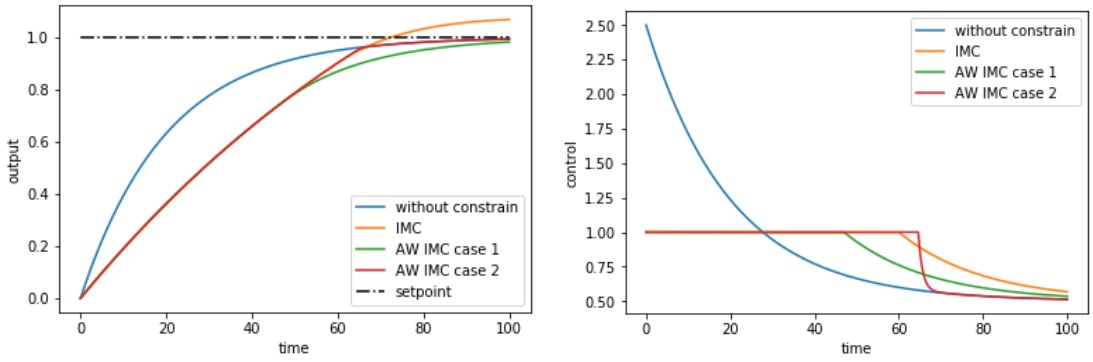


Figure 4.6: Plot output and input versus time where the desired value or setpoint is equal to 1.

Chapter 5

Anti-windup input-output linearization scheme for SISO systems

The main purpose of this chapter is to find the input $\tilde{u}$ of a single-input single output (SISO) nonlinear system control problem with constraints such that the output y reaches and stabilizes at a known desirable constant value y^* and is to close to y^{wc} as possible at each instant of time. This will be solved by linearizing the SISO system in a linear system and finally we will apply the anti-windup technique to this linear system studied in the previous chapter. This was done in the article [5] and we are going to do that with more details and give the implicit solution of an example.

5.1 Notations

The Lie derivative or directional derivative is defined as follows:

$$\begin{array}{rccl} L_f h: & \mathbb{R}^n & \longrightarrow & \mathbb{R} \\ & x & \mapsto & \langle \dfrac{\partial h}{\partial x}(x), f(x) \rangle = \displaystyle\sum_{i=1}^{n} \frac{\partial h}{\partial x_i}(x) f_i(x) \end{array}$$

where $f(x) = [f_1(x), \ldots, f_n(x)] \in \mathbb{R}^n$ and $h : \mathbb{R}^n \to \mathbb{R}$ is differentiable. The state-dependent saturation operator for a signal u is defined as:

$$sat(u, x(t)) := \begin{cases} u^{max}(x(t)) & \text{if } u > u^{max}(x(t)) \\ u^{min}(x(t)) & \text{if } u < u^{min}(x(t)) \\ u(t) & \text{otherwise} \end{cases}$$

5.2 Anti-windup feedback linearizing design

Our nonlinear state-space equation is described as follows:

$$\left.\begin{array}{lcl} \dot{x} & = & f(x(t)) + g(x(t))u(t) \\ y(t) & = & h(x(t)) \\ u^{min} & \leq u(t) \leq & u^{max} \end{array}\right\} \tag{5.1}$$

where the functions $f, g, h : \mathbb{R}^n \to \mathbb{R}^n$. The equation (5.1) is called SISO systems.

We define the relative degree r at the point x_0 as the integer r which satisfies:

$$\begin{array}{l} L_g L_f^{i-1} h(x) = 0 \quad 1 \leq i \leq r-1, \text{ for all } x \text{ near to } x_0 \\ L_g L_f^{r-1} h(x) \neq 0 \quad \text{for all } x \text{ near to } x_0. \end{array}$$

When $r = 1$, we have $L_g h(x) \neq 0$ for all x near to x_0. Using equation (5.1), we have the following:

$$\begin{array}{rcl} \dot{y} & = & \dfrac{\partial h}{\partial x}(x)(f(x) + g(x)u) \\ \dot{y} & = & L_f h(x) + L_g h(x) u \end{array}$$

When $r = 2$, we have $L_g h(x) = 0$ and $L_g L_f h(x) \neq 0$ for all x near to x_0. Using equation (5.1), we have the following:

$$\begin{array}{rcl} \dot{y} & = & L_f h(x) \\ \ddot{y} & = & L_f^2 h(x) + L_g L_f h(x) u. \end{array}$$

In general case, when the relative degree is r, we have

$$\begin{array}{rcl} y^{(i)} & = & L_f^i h(x) \quad 0 \leq i \leq r-1 \\ y^{(r)} & = & L_f^r h(x) + L_g L_f^{r-1} h(x) u. \end{array} \tag{5.2}$$

The main purpose of this section is to transform the nonlinear system (5.1) in a linear system with control v.

Let P be a linear plant with input v and output y,

$$P(s) = \frac{1}{\sum_{i=0}^{n} \lambda_i s^i}, \quad \lambda_n = 1. \tag{5.3}$$

The equation (5.3) is equivalent to,

$$y^{(r)} = v - \sum_{i=0}^{r-1} \lambda_i y^{(i)} \tag{5.4}$$

Using the equation (5.2) and (5.4), we have the following:

$$\begin{array}{rcl} v & = & L_f^r h(x) + L_g L_f^{r-1} h(x) u + \sum_{i=0}^{r-1} \lambda_i y^{(i)} \\ u & = & -\dfrac{L_f^r h(x)}{L_g L_f^{r-1} h(x)} + (v - \sum_{i=0}^{r-1} \lambda_i y^{(i)}) \dfrac{1}{L_g L_f^{r-1} h(x)} \end{array}$$

$$\boxed{u = \alpha(x) + (v - \sum_{i=0}^{r-1} \lambda_i y^{(i)})\beta(x)} \tag{5.5}$$

where

$$\begin{aligned} \alpha(x) &:= -\frac{L_f^r h(x)}{L_g L_f^{r-1} h(x)} \\ \beta(x) &:= \frac{1}{L_g L_f^{r-1} h(x)}. \end{aligned}$$

The new control v is satured,

$$u^{min} \leq \alpha(x) + (v - \sum_{i=0}^{r-1} \lambda_i y^{(i)})\beta(x) \leq u^{max}$$

$$\frac{u^{min} - \alpha(x)}{\beta(x)} + \sum_{i=0}^{r-1} \lambda_i y^{(i)} \leq v \leq \frac{u^{max} - \alpha(x)}{\beta(x)} + \sum_{i=0}^{r-1} \lambda_i y^{(i)}$$

$$\boxed{v^{min}(x(t)) \leq v \leq v^{max}(x(t))} \tag{5.6}$$

where

$$\begin{aligned} v^{min}(x(t)) &:= \frac{u^{min} - \alpha(x)}{\beta(x)} + \sum_{i=0}^{r-1} \lambda_i y^{(i)} \\ v^{max}(x(t)) &:= \frac{u^{max} - \alpha(x)}{\beta(x)} + \sum_{i=0}^{r-1} \lambda_i y^{(i)}. \end{aligned}$$

The equation (5.4) and the inequality (5.6) is equivalent to a linear system with constraints being v the new input and y the output where v is limited, so an anti-windup scheme for linear systems who was studied in the previous chapter will be applied to obtained v and y. Using the equation (5.5), the input u is obtained.

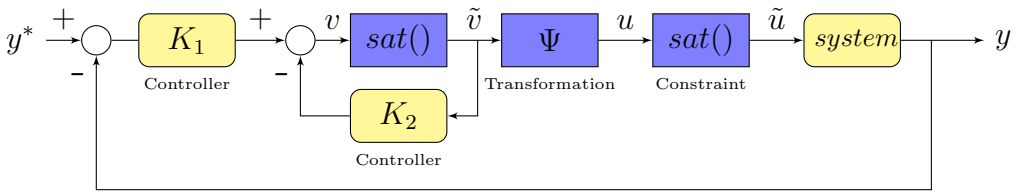


Figure 5.1: Scheme of anti-windup input-output feedback linearizing controller.

5.3 Example

We consider the following nonlinear SISO system:

$$\begin{aligned} \dot{x} &= 0.01e^{x}(-x+2u) \\ y &= x \end{aligned}$$

subject to the constraint $-1 \leq u \leq 1$ and the setpoint y^* is equal to 1.

The linear system is taken from the studies of Zheng [4]:

$$P(s) = \frac{2}{100s+1}.$$

The IMC controller designed,

$$K = \frac{100s+1}{40s}$$

The filter is $F(s) = 50(s+1)$, then the anti-windup controllers are:

$$K_1 = \frac{50(s+1)}{20s+1} \quad \text{and} \quad K_2 = \frac{1880s-1}{(100s+1)(20s+1)}.$$

5.3.1 Close loop

a) Unconstrain system is obtained by the the scheme in Figure 5.2:

$$\begin{aligned} Y &= PV \\ V &= K(Y^* - Y) \end{aligned}$$

The mathematical equations are the following:

$$\begin{aligned} \dot{y} &= -0.01y + 0.02u \\ u &= 2.5(y^* - y) + z \end{aligned}$$

where

$$\begin{aligned} \dot{z} &= 0.025(y^* - y) \\ [y(0), z(0)] &= [0, 0]. \end{aligned}$$

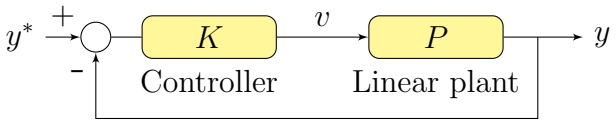


Figure 5.2: Unconstrain linear system scheme.

b) Conventional linear IMC system is obtained by the scheme in Figure 5.3:

$$\begin{aligned} Y &= syst(\tilde{u}) \\ \tilde{u} &= sat(u) \\ U &= K(Y^* - Y) \end{aligned}$$

The mathematical equations are the following:

$$\begin{aligned} \dot{y} &= -0.01e^{y}(-y+2\tilde{u}) \\ \tilde{u} &= sat(u) \\ u &= 2.5(y^{*}-y)+z \end{aligned}$$

where

$$\begin{aligned} \dot{z} &= 0.025(y^{*}-y) \\ [y(0),z(0)] &= [0,0] \end{aligned}$$

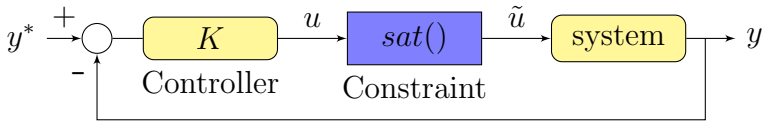


Figure 5.3: Conventional linear IMC scheme.

c) Linearization systems is obtained by the scheme in Figure 5.4:

$$\begin{aligned} y &= syst(\tilde{u}) \\ \tilde{u} &= sat(u) \\ u &= \Psi(\tilde{v}) \\ V &= K(Y^{*}-Y) \\ \tilde{v} &= sat(v,x) \end{aligned}$$

The mathematical equations are the following:

$$\begin{aligned} \dot{y} &= -0.01e^{y}(-y+2\tilde{u}) \\ \tilde{u} &= sat(u) \\ u &= \frac{2\tilde{v}-y+ye^{y}}{2e^{y}} \\ \tilde{v} &= sat(v,x) \\ v &= 2.5(y^{*}-y)+z \\ v^{min} &= u^{min}e^{y}+\tfrac{1}{2}y(1-e^{y}) \\ v^{max} &= u^{max}e^{y}+\tfrac{1}{2}y(1-e^{y}) \end{aligned}$$

where

$$\begin{aligned} \dot{z} &= 0.025(y^{*}-y) \\ [y(0),z(0)] &= [0,0]. \end{aligned}$$

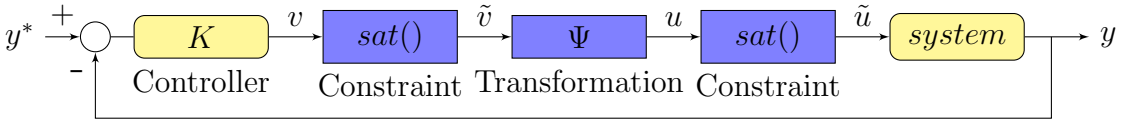


Figure 5.4: Linearizing scheme.

d) Anti-windup IMC system is obtained by the scheme in Figure 5.5:

$$\begin{aligned} Y &= syst(\tilde{u}) \\ \tilde{u} &= sat(u) \\ U &= K_1(Y^* - Y) - K_2\tilde{U} \end{aligned}$$

The mathematical equations are the following:

$$\begin{aligned} \dot{y} &= -0.01e^y(-y + 2\tilde{u}) \\ \tilde{u} &= sat(u) \\ u &= 2.5(y^* - y) + \tfrac{95}{2}z_1 + \tfrac{99}{4}z_2 - \tfrac{95}{4}z_3 \end{aligned}$$

where

$$\begin{aligned} \dot{z_1} &= 0.05(y^* - y - z_1) \\ \dot{z_2} &= 0.01(\tilde{u} - z_2) \\ \dot{z_3} &= 0.05(\tilde{u} - z_3) \\ [y(0), z_1(0), z_2(0), z_3(0)] &= [0, 0, 0, 0]. \end{aligned}$$

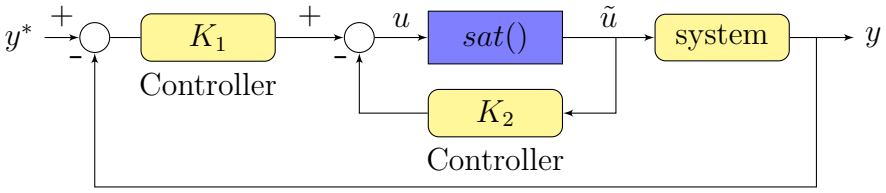


Figure 5.5: Anti-windup linear IMC scheme.

e) Anti-windup linearizing system is obtained by the scheme in Figure 5.1:

$$\begin{aligned} y &= syst(\tilde{u}) \\ \tilde{u} &= sat(u) \\ u &= \Psi(\tilde{v}) \\ V &= K_1(Y^* - Y) - K_2\tilde{V} \\ \tilde{v} &= sat(v, x) \end{aligned}$$

The mathematical equations are the following:

$$\begin{aligned} \dot{y} &= -0.01e^y(-y + 2\tilde{u}) \\ \tilde{u} &= sat(u) \\ u &= \frac{2\tilde{v} - y + ye^y}{2e^y} \\ \tilde{v} &= sat(v, x) \\ v &= 2.5(y^* - y) + \tfrac{95}{2}z_1 + \tfrac{99}{4}z_2 - \tfrac{95}{4}z_3 \\ v^{min} &= u^{min}e^y + \tfrac{1}{2}y(1 - e^y) \\ v^{max} &= u^{max}e^y + \tfrac{1}{2}y(1 - e^y) \end{aligned}$$

where

$$\begin{aligned} \dot{z_1} &= 0.05(y^* - y - z_1) \\ \dot{z_2} &= 0.01(\tilde{v} - z_2) \\ \dot{z_3} &= 0.05(\tilde{v} - z_3) \\ [y(0), z_1, z_2, z_3] &= [0, 0, 0, 0]. \end{aligned}$$

5.3.2 Simulation results

We present the simulation results of the input and output of the Example 5.3 without using the anti-windup technique.

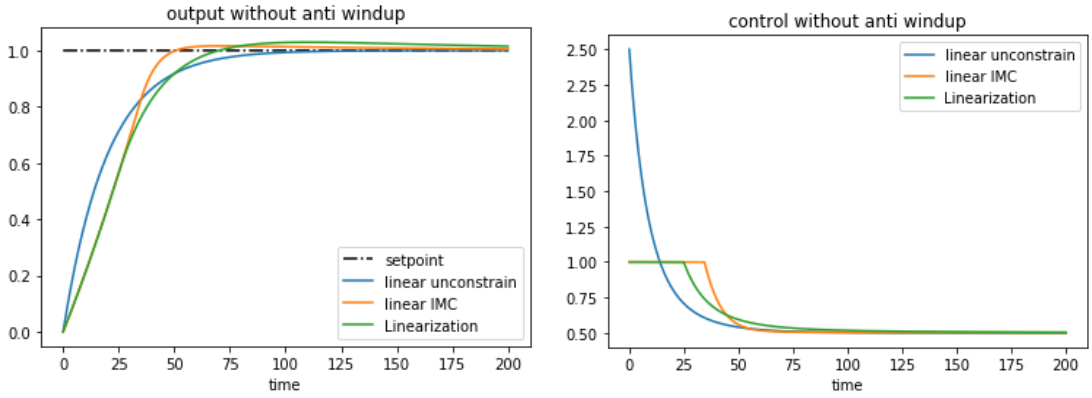


Figure 5.6: a) Unconstrain solution. b) IMC. c) Input-output linearization. The desired value or setpoint is equal to 1.

We present the simulation results of the input and output of the Example 5.3 using the anti-windup technique.

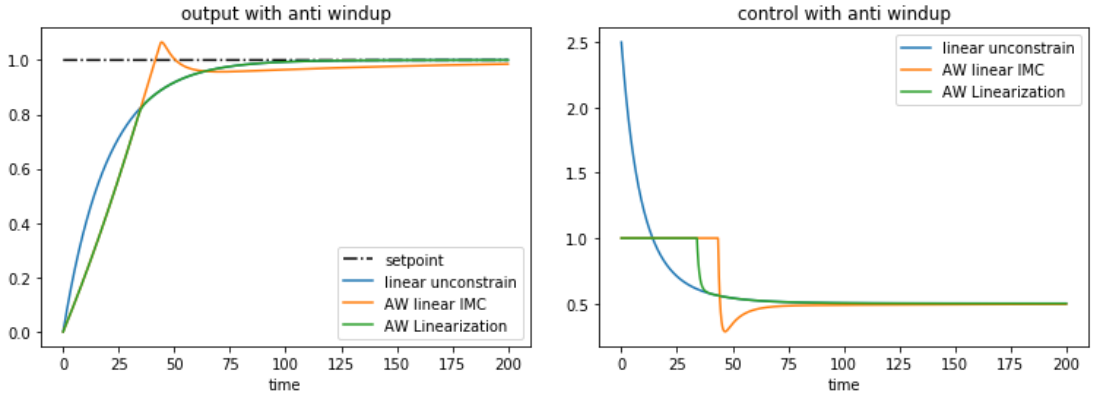


Figure 5.7: a) Unconstrain solution. d) Anti-windup IMC. e) Anti-windup input-output Linearization. The desired value or setpoint is equal to 1.

Chapter 6

An anti-windup scheme for multivariable nonlinear systems

The main purpose of this chapter is to find the input $\tilde{u}$ of a multi-input multi-output (MIMO) nonlinear system control problem with constraints such that the output y is to close to y^{wc} at each instant of time. The solution of this control problem is a generalization of the case SISO nonlinear system control problem with constraints. It was done in the article [6] and we are going to do that with more details and give the implicit solution of an example.

6.1 Problem formulation

In this chapter, the state-space system is a multivariable nonlinear systems,

$$\left.\begin{array}{lcll} \dot{x} & = & f(x(t)) + g(x(t))u(t) & \\ y(t) & = & h(x(t)) & \\ u_i^{min} & \leq u_i(t) \leq & u_i^{max} \quad \forall i = 1, ..., m & \end{array}\right\} \tag{6.1}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$, $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$, $h : \mathbb{R}^n \to \mathbb{R}^m$, $u := [u_1, \ldots, u_m]$.

The main purpose is to keep the output of the constrained system as close to the output of the unconstrained system as possible in each instant of time.

6.2 Nonlinear anti-windup design

6.2.1 Relative degree

Many reasoning used to define the concepts for multivariable systems are an extension of the SISO case. In the multivariable case, the notion of relative degree is extended by

- The vector of relative degrees $[r_1, ..., r_m]$ defined in a neighbourhood of x° by

$$[L_{g_1} L_f^k h_i(x), ..., L_{g_m} L_f^k h_i(x)] = 0 \quad 1 \leq k < r_i - 1 \quad i = 1, ..., m \tag{6.2}$$

$$[L_{g_1}L_f^{r_i-1}h_i(x),...,L_{g_m}L_f^{r_i-1}h_i(x)] \neq 0 \quad i=1,...,m \tag{6.3}$$

where $g_i : \mathbb{R}^n \to \mathbb{R}^n$ for all $i = 1, \ldots, m$ and $g = [g_1, \ldots, g_m]$.

- The matrix $\beta(x)$ defined by

$$\beta(x) \;:=\; \begin{bmatrix} L_{g_1}L_f^{r_1-1}h_1(x) & \ldots & L_{g_m}L_f^{r_1-1}h_1(x) \\ \vdots & & \vdots \\ L_{g_1}L_f^{r_m-1}h_m(x) & \ldots & L_{g_m}L_f^{r_m-1}h_m(x) \end{bmatrix},$$

where $h_i : \mathbb{R}^n \to \mathbb{R}$ for all $i = 1, \ldots, m$ and $h = [h_1, \ldots, h_m]$.

Using the equations (6.1), (6.2) and (6.3), we have the following:

$$y_i^{(k)} = L_f^k h_i(x) \quad k=0,...,r_i-1 \quad i=1,...,m \tag{6.4}$$

$$y_i^{(r_i)} = L_f^{r_i}h_i(x) + [L_{g_1}L_f^{r_i-1}h_i(x),...,L_{g_m}L_f^{r_i-1}h_i(x)]u \quad i=1,...,m \tag{6.5}$$

6.2.2 Input-output linearization design

The main purpose of multivariable input-output linearization is the design of a transformation $u = \Psi(x, v)$ such that the relationship between the output y and the transformed input v is a linear system. So that, we present the following linear system:

$$a_{ir_i}y_i^{(r_i)} = v_i - \sum_{k=0}^{r_i-1} a_{ik}y_i^{(k)} \quad i=1,\ldots,m \tag{6.6}$$

or equivalent to

$$Y_i = \frac{V_i}{\sum_{k=0}^{r_i} a_{ik}s^k} \quad i=1,\ldots,m$$

where v_i is the ith component of v, Y_i and V_i are respectively the Laplace transforms of y_i and v_i.

Using (6.4) and (6.5), the equation (6.6) is equivalent as follows,

$$a_{ir_i}L_f^{r_i}h_i(x) + a_{ir_i}[L_{g_1}L_f^{r_i-1}h_i(x),...,L_{g_m}L_f^{r_i-1}h_i(x)]u = v_i - \sum_{k=0}^{r_i-1} a_{ik}L_f^k h_i(x) \;\; i=1,...,m$$

Then,

$$a_{ir_i}[L_{g_1}L_f^{r_i-1}h_i(x),...,L_{g_m}L_f^{r_i-1}h_i(x)]u = v_i - \sum_{k=0}^{r_i} a_{ik}L_f^k h_i(x) \quad i=1,...,m$$

The previous equation can be expressed as follows,

$$A\beta(x)u = v - b \tag{6.7}$$

where

$$A := \begin{bmatrix} a_{1r_1} & 0 & \dots & 0 \\ 0 & a_{2r_2} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & a_{mr_m} \end{bmatrix}$$

$$v := [v_1, ..., v_m]^T$$

$$b(x) := \begin{bmatrix} \sum_{k=0}^{r_1} a_{1k} L_f^k h_1(x) \\ \vdots \\ \sum_{k=0}^{r_m} a_{mk} L_f^k h_m(x) \end{bmatrix}$$

Therefore, the equation (6.7) is equivalent to the next equation searched,

$$\boxed{u = \Psi(x, v)}$$

where

$$\boxed{\Psi(x, v) := \beta^{-1}(x) A^{-1}(v - b)} \tag{6.8}$$

6.3 Example

Consider the following MIMO nonlinear system control problem:

$$\begin{array}{rcl} \dot{x_1} & = & -0.1(1 - e^{x_1}) + 0.4u_1 - 0.5u_2 \\ \dot{x_2} & = & -0.2x_2 - \frac{0.1x_2}{1+x_2} - 0.3u_1 + 0.4u_2 \\ y_1 & = & x_1 \\ y_2 & = & x_2 \\ -15 & < u_i < & 15 \quad i = 1, 2 \end{array}$$

The setpoints y_1^* and y_2^* are respectively 0.61 and 0.79.

The previous MIMO nonlinear system is equivalent to a state-space system like (6.1), where

$$\begin{array}{rcl} f(x) & = & \left[-0.1(1 - e^{x_1}), -0.2x_2 - 0.1\dfrac{x_2}{1 + x_2}\right]^T \\ g(x) & = & \begin{bmatrix} 0.4 & -0.5 \\ -0.3 & 0.4 \end{bmatrix} \\ h(x) & = & x \\ u^{min} & = & [-15, -15]^T \\ u^{max} & = & [15, 15]^T \end{array}$$

The relative degree defined in the Subsection 6.2.1 of this system is $[1, 1]$, because

$$\begin{aligned}[L_{g_1}h_1(x), L_{g_2}h_1(x)] &= [0.4, -0.5] \\ [L_{g_1}h_2(x), L_{g_2}h_2(x)] &= [-0.3, 0.4].\end{aligned}$$

Then the matrix $\beta(x)$ defined in the Subsection 6.2.1 is

$$\beta(x) = \begin{bmatrix} 0.4 & -0.5 \\ -0.3 & 0.4 \end{bmatrix}.$$

We propose the following linear system:

$$\begin{aligned}Y_1 &= \frac{1}{1+10s}(4V_1 - 5V_2) \\ Y_2 &= \frac{1}{1+10s}(-3V_1 + 4V_2).\end{aligned}$$

The previous equations are equivalent to,

$$Y = P_L V$$

where

$$\begin{aligned}P_L &:= \frac{1}{1+10s}\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} \\ Y &= [Y_1, Y_2]^T \\ V &= [V_1, V_2]^T.\end{aligned}$$

Then the matrices A and $b(x)$ defined in the Subsection 6.2.2 are

$$A = 10I_2$$

and

$$b(x) = \begin{bmatrix} x_1 - (1 - e^{x_1}) \\ -x_2 - \dfrac{x_2}{1+x_2} \end{bmatrix}$$

because

$$\begin{aligned}[L_f^0 h_1(x), L_f^1 h_1(x)] &= [x_1, -0.1(1 - e^{x_1})] \\ \left[L_f^0 h_2(x), L_f^1 h_2(x)\right] &= [x_2, -0.2x_2 - 0.1\frac{x_2}{1+x_2}] \\ [a_{10}, a_{11}] &= [1, 10] \\ [a_{20}, a_{21}] &= [1, 10].\end{aligned}$$

Using the equation (6.8), we have the transformation searched and new control constrains defined in the Subsection 6.2.2,

$$u = \Psi(x, v)$$

where

$$\begin{aligned}
\Psi(x,v) &= \begin{bmatrix} v_1 + 4(1-e^{x_1}-x_1) + 5x_2\dfrac{2+x_2}{1+x_2} \\ \\ v_2 + 3(1-e^{x_1}-x_1) + 4x_2\dfrac{2+x_2}{1+x_2} \end{bmatrix} \\
v^{min} &= [-15 - 4(1-e^{x_1}-x_1) - 5x_2\frac{(2+x_2)}{1+x_2}, -15 - 3(1-e^{x_1}-x_1) - 4x_2\frac{(2+x_2)}{1+x_2}]^T \\
v^{max} &= [15 - 4(1-e^{x_1}-x_1) - 5x_2\frac{(2+x_2)}{1+x_2}, 15 - 3(1-e^{x_1}-x_1) - 4x_2\frac{(2+x_2)}{1+x_2}]^T.
\end{aligned}$$

We propose the following IMC controller K for the previous linear system:

$$K = (10 + \frac{1}{s}) \begin{bmatrix} \frac{4}{3} & \frac{5}{3} \\ \\ 1 & \frac{4}{3} \end{bmatrix}$$

Using the filter $f = 2.5(s+1)I_2$ and the anti-windup technique in Section 4.3, the anti-windup controllers K_1 and K_2 are:

$$\begin{aligned}
K_1 &= \tfrac{5}{2}\tfrac{s+1}{3s+1}I_2 = \tfrac{5}{6}I_2 + \tfrac{5}{3}\tfrac{1}{3s+1}I_2 \\
K_2 &= \tfrac{1}{2}\tfrac{1}{10s+1}\tfrac{1}{3s+1}\begin{bmatrix} 34s-2 & -25(s+1)(3s+2) \\ -15(s+1)(3s+2) & 34s-2 \end{bmatrix} \\
&= -\begin{bmatrix} 0 & \frac{5}{4} \\ \\ \frac{3}{4} & 0 \end{bmatrix} - \tfrac{1}{10s+1}\begin{bmatrix} \frac{27}{7} & \frac{765}{28} \\ \\ \frac{459}{28} & \frac{27}{7} \end{bmatrix} + \tfrac{1}{3s+1}\begin{bmatrix} \frac{20}{7} & \frac{25}{7} \\ \\ \frac{15}{7} & \frac{20}{7} \end{bmatrix}
\end{aligned}$$

6.3.1 Close loop

We present the implicit mathematical solution using the anti-windup linearizing scheme. Before to do that, it is neccesary to follow a sequence of steps: linear scheme without constraints, nonlinear scheme without constraints, anti-windup linearizing scheme without constraints and finally, the anti-windup linearizing scheme with constraints.

a) Linear system without constraints:

$$\begin{aligned}
Y &= P_L V \\
V &= K(Y^* - Y)
\end{aligned} \tag{6.9}$$

the equation (6.9) is equivalent to:

$$\begin{aligned}
V_1 &= \tfrac{40}{3}(Y_1^* - Y_1) + \tfrac{50}{3}(Y_2^* - Y_2) + \tfrac{4}{3}W_1 + \tfrac{5}{3}W_2 \\
V_2 &= 10(Y_1^* - Y_1) + \tfrac{40}{3}(Y_2^* - Y_2) + W_1 + \tfrac{4}{3}W_2
\end{aligned}$$

where

$$\begin{aligned} W_1 &= \tfrac{1}{s}(Y_1^* - Y_1) \\ W_2 &= \tfrac{1}{s}(Y_2^* - Y_2). \end{aligned}$$

The implicit solution of the control problem of this item is:

$$\begin{aligned} \dot{y}_1 &= -0.1y_1 + 0.4v_1 - 0.5v_2 \\ \dot{y}_2 &= -0.1y_2 - 0.3v_1 + 0.4v_2 \\ v_1 &= \tfrac{40}{3}(y_1^* - y_1) + \tfrac{50}{3}(y_2^* - y_2) + \tfrac{4}{3}w_1 + \tfrac{5}{3}w_2 \\ v_2 &= 10(y_1^* - y_1) + \tfrac{40}{3}(y_2^* - y_2) + w_1 + \tfrac{4}{3}w_2 \end{aligned}$$

where

$$\left.\begin{aligned} \dot{w}_1 &= y_1^* - y_1 \\ \dot{w}_2 &= y_2^* - y_2 \\ [y_1(0), y_2(0), w_1(0), w_2(0)] &= [0,0,0,0]. \end{aligned}\right\} \tag{6.10}$$

b) Nonlinear system without constraints:

$$\begin{aligned} y &= system(u) \\ u &= \Psi(x, v) \\ V &= K(Y^* - Y) \end{aligned}$$

The implicit solution of the control problem of this item is:

$$\begin{aligned} \dot{y}_1 &= -0.1(1 - e^{y_1}) + 0.4u_1 - 0.5u_2 \\ \dot{y}_2 &= -0.2y_2 - 0.1\tfrac{y_2}{1+y_2} - 0.3u_1 + 0.4u_2 \\ u_1 &= v_1 + 4(1 - e^{x_1} - x_1) + 5x_2\tfrac{x_2+2}{x_2+1} \\ u_2 &= v_2 + 3(1 - e^{x_1} - x_1) + 4x_2\tfrac{x_2+2}{x_2+1} \\ v_1 &= \tfrac{40}{3}(y_1^* - y_1) + \tfrac{50}{3}(y_2^* - y_2) + \tfrac{4}{3}w_1 + \tfrac{5}{3}w_2 \\ v_2 &= 10(y_1^* - y_1) + \tfrac{40}{3}(y_2^* - y_2) + w_1 + \tfrac{4}{3}w_2 \end{aligned}$$

where w_1 and w_2 satisfy the ordinary differential equations in (6.10).

c) Anti-windup linearizing system without constraints:

$$\begin{aligned} y &= system(u) \\ u &= \Psi(x, v) \\ V &= K_1(Y^* - Y) - K_2\tilde{V} \\ V &= \tilde{V} \end{aligned} \tag{6.11}$$

The equation (6.11) is equivalent to:

$$\begin{aligned} V_1 &= \tfrac{5}{6}(Y_1^* - Y_1) + \tfrac{5}{3}\tfrac{1}{3s+1}(Y_1^* - Y_1) + \tfrac{5}{4}\tilde{V}_2 + \tfrac{27}{7}\tfrac{1}{10s+1}\tilde{V}_1 + \tfrac{765}{28}\tfrac{1}{10s+1}\tilde{V}_2 - \tfrac{20}{7}\tfrac{1}{3s+1}\tilde{V}_1 \\ &\quad -\tfrac{25}{7}\tfrac{1}{3s+1}\tilde{V}_2 \\ V_2 &= \tfrac{5}{6}(Y_2^* - Y_2) + \tfrac{5}{3}\tfrac{1}{3s+1}(Y_2^* - Y_2) + \tfrac{3}{4}\tilde{V}_1 + \tfrac{459}{28}\tfrac{1}{10s+1}\tilde{V}_1 + \tfrac{27}{7}\tfrac{1}{10s+1}\tilde{V}_2 - \tfrac{15}{7}\tfrac{1}{3s+1}\tilde{V}_1 \\ &\quad -\tfrac{20}{7}\tfrac{1}{3s+1}\tilde{V}_2 \end{aligned}$$

The previous equations are equivalent to:

$$\begin{aligned} V_1 &= \tfrac{5}{6}(Y_1^* - Y_1) + \tfrac{5}{3}W_1 + \tfrac{5}{4}\tilde{V}_2 + \tfrac{27}{7}X_1 + \tfrac{765}{28}X_2 - \tfrac{20}{7}Z_1 - \tfrac{25}{7}Z_2 \\ V_2 &= \tfrac{5}{6}(Y_2^* - Y_2) + \tfrac{5}{3}W_2 + \tfrac{3}{4}\tilde{V}_1 + \tfrac{459}{28}X_1 + \tfrac{27}{7}X_2 - \tfrac{15}{7}Z_1 - \tfrac{20}{7}Z_2 \end{aligned}$$

where

$$\begin{aligned} W_1 &= \tfrac{1}{3s+1}(Y_1^* - Y_1) \\ W_2 &= \tfrac{1}{3s+1}(Y_2^* - Y_2) \\ X_1 &= \tfrac{1}{10s+1}\tilde{V}_1 \\ X_2 &= \tfrac{1}{10s+1}\tilde{V}_2 \\ Z_1 &= \tfrac{1}{3s+1}\tilde{V}_1 \\ Z_2 &= \tfrac{1}{3s+1}\tilde{V}_2 \end{aligned}$$

The implicit solution of the control problem of this item is:

$$\begin{aligned} \dot{y}_1 &= -0.1(1 - e^{y_1}) + 0.4u_1 - 0.5u_2 \\ \dot{y}_2 &= -0.2y_2 - 0.1\tfrac{y_2}{1+y_2} - 0.3u_1 + 0.4u_2 \\ u_1 &= v_1 + 4(1 - e^{x_1} - x_1) + 5x_2\tfrac{x_2+2}{x_2+1} \\ u_2 &= v_2 + 3(1 - e^{x_1} - x_1) + 4x_2\tfrac{x_2+2}{x_2+1} \\ v_1 - \tfrac{5}{4}v_2 &= \tfrac{5}{6}(y_1^* - y_1) + \tfrac{5}{3}w_1 + \tfrac{27}{7}x_1 + \tfrac{765}{28}x_2 - \tfrac{20}{7}x_1 - \tfrac{25}{7}z_2 \\ -\tfrac{3}{4}v_1 + v_2 &= \tfrac{5}{6}(y_2^* - y_2) - \tfrac{5}{3}w_2 + \tfrac{459}{28}x_1 + \tfrac{27}{7}x_2 - \tfrac{15}{7}z_1 - \tfrac{20}{7}z_2 \end{aligned}$$

where

$$\left.\begin{array}{l} \dot{w}_1 = \tfrac{1}{3}(y_1^* - y_1 - w_1) \\ \dot{w}_2 = \tfrac{1}{3}(y_2^* - y_2 - w_2) \\ \dot{x}_1 = \tfrac{1}{10}(v_1 - x_1) \\ \dot{x}_2 = \tfrac{1}{10}(v_2 - x_2) \\ \dot{z}_1 = \tfrac{1}{3}(v_1 - z - 1) \\ \dot{z}_2 = \tfrac{1}{3}(v_2 - z_2) \\ [y_1, y_2, w_1, w_2, x_1, x_2, z_1, z_2]\,(0) = [0, 0, 0, 0, 0, 0, 0, 0]. \end{array}\right\} \quad (6.12)$$

d) Anti-windup linearizing system with constraints:

$$\begin{aligned} y &= system(\tilde{u}) \\ \tilde{u} &= sat(u) \\ u &= \Psi(\tilde{v}) \\ V &= K_1(Y^* - Y) - K_2\tilde{V} \\ \tilde{v} &= sat(v, x) \end{aligned} \quad (6.13)$$

The implicit solution of the control pronlem of this item is:

$$
\begin{aligned}
\dot{y}_1 &= -0.1(1-e^{y_1}) + 0.4\tilde{u_1} - 0.5\tilde{u_2} \\
\dot{y}_2 &= -0.2y_2 - 0.1\tfrac{y_2}{1+y_2} - 0.3\tilde{u_1} + 0.4\tilde{u_2} \\
u_1 &= v_1 + 4(1-e^{x_1}-x_1) + 5x_2\tfrac{x_2+2}{x_2+1} \\
u_2 &= v_2 + 3(1-e^{x_1}-x_1) + 4x_2\tfrac{x_2+2}{x_2+1} \\
\tilde{u}_1 &= sat(u_1, u_1^{min}, u_1^{max}) \\
\tilde{u}_2 &= sat(u_2, u_2^{min}, u_2^{max}) \\
v_1 &= \tfrac{5}{6}(y_1^* - y_1) + \tfrac{5}{3}w_1 + \tfrac{5}{4}\tilde{v}_2 + \tfrac{27}{7}x_1 + \tfrac{765}{28}x_2 - \tfrac{20}{7}x_1 - \tfrac{25}{7}z_2 \\
v_2 &= \tfrac{5}{6}(y_2^* - y_2) - \tfrac{5}{3}w_2 + \tfrac{3}{4}\tilde{v}_1 + \tfrac{459}{28}x_1 + \tfrac{27}{7}x_2 - \tfrac{15}{7}z_1 - \tfrac{20}{7}z_2 \\
\tilde{v}_1 &= sat(v_1, v_1^{min}, v_1^{max}) \\
\tilde{v}_2 &= sat(v_2, v_2^{min}, v_2^{max})
\end{aligned}
$$

where u_i^{min}, u_i^{max}, v_i^{min} and v_i^{max} were defined at the beginning of this section.

6.3.2 Simulation results

The following graphics describe the input and output of the control problems of the previous subsection.

a) Linear system without constraints:

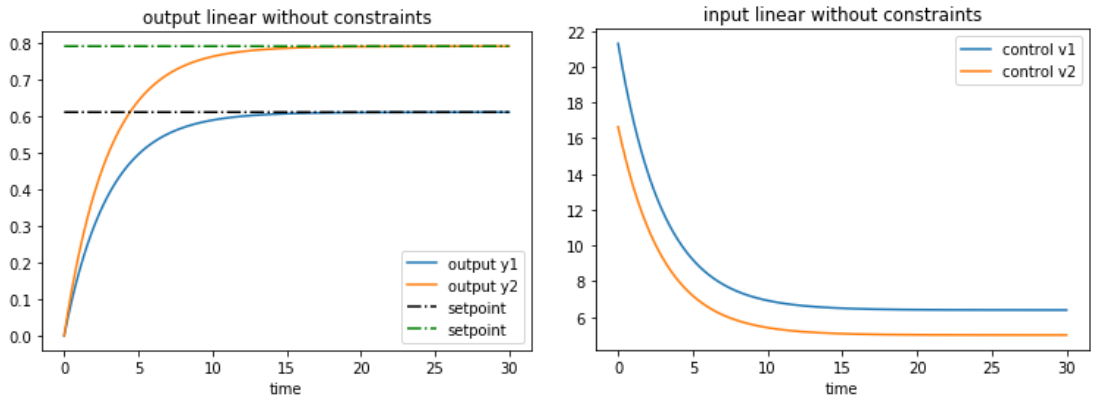


Figure 6.1: Linear system solution without constraints.

b) Nonlinear system without constraints:

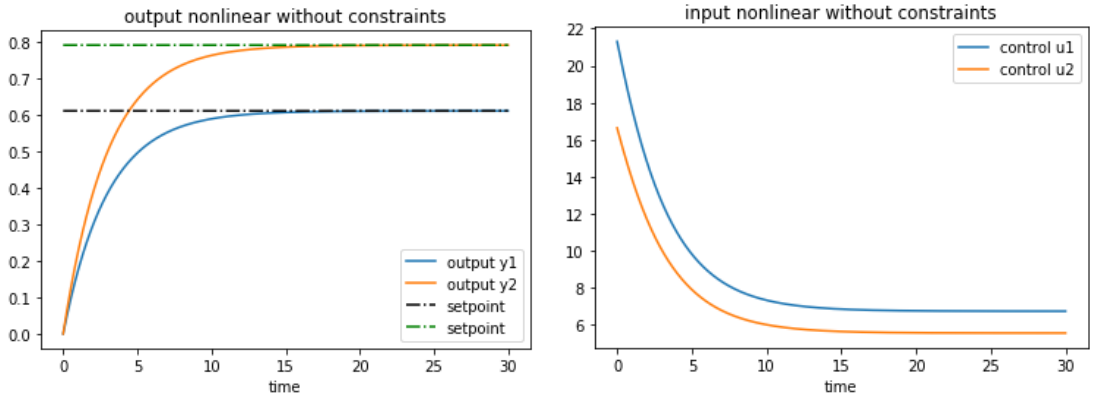


Figure 6.2: Nonlinear system solution without constraints.

c) Anti-windup linearizing nonlinear system without constraints:

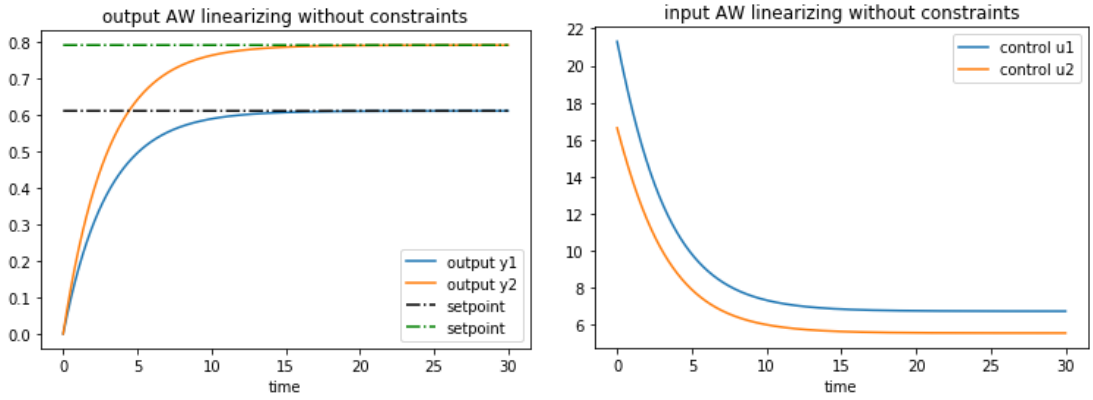


Figure 6.3: Anti-windup linearizing nonlinear system solution without constraints.

d) Anti-windup linearizing nonlinear system with constraints:

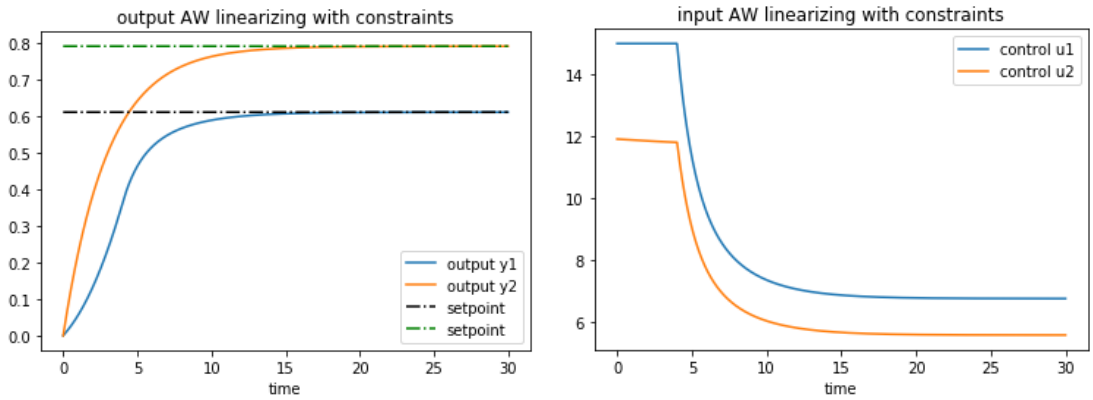


Figure 6.4: Anti-windup linearizing nonlinear system solution with constraints.

Chapter 7

Control theory applied to the wine fermentation process with oenological condition

The main purpose of this chapter is to control the wine fermentation with oenological condition represented for the mathematical model (3.3), that is to say, we are going to add and remove the chemical concentrations of the continuous bioreactor with certain velocities Q_1 and Q_2 such that the sugar S and the rate production VCO_2 reaches and stabilizes at the desired values S^* and VCO_2^* respectively.

7.1 Relative degree

The relative degree defined in the Subsection 6.2.1 of the dynamical system (3.3) is $[1, 1]$, because

$$\begin{aligned}[L_{g_1}h_1(\xi), L_{g_2}h_1(\xi)] &= [\beta_{11}(\xi), \beta_{12}(\xi)] \\ [L_{g_1}h_2(\xi), L_{g_2}h_2(\xi)] &= [\beta_{21}(\xi), \beta_{22}(\xi)],\end{aligned}$$

where $\xi = [X, N, Tr, S, E]^t$, the functions h_i and g_i for $i = 1, 2$ are respectively the columns of the matrix functions h and g defined in (3.3),

$$\begin{aligned}\beta_{11}(\xi) &:= \frac{S_{in} - S}{V} \\ \beta_{12}(\xi) &:= -\frac{S}{V} \\ \beta_{21}(\xi) &:= \frac{1}{2.17}\frac{Tr}{V}\left[\frac{K_S(S_{in} - S) + K_{SI}\alpha_S S^2 E^{\alpha_S}}{(S + K_S + K_{SI}SE^{\alpha_S})^2} - \nu(E, S)\right] \\ \beta_{22}(\xi) &:= \frac{1}{2.17}\frac{Tr}{V}\left[\frac{K_{SI}\alpha_S S^2 E^{\alpha_S} - K_S S}{(S + K_S + K_{SI}SE^{\alpha_S})^2} - \nu(E, S)\right].\end{aligned}$$

Then the matrix $\beta(x)$ defined in the Subsection 6.2.1 is

$$\beta(x) = \begin{bmatrix}\beta_{11}(\xi) & \beta_{12}(\xi) \\ \beta_{21}(\xi) & \beta_{22}(\xi)\end{bmatrix}.$$

7.2 Linearizing control law

We propose the following linear system:

$$\begin{aligned} \dot{y}_1 &= v_1 \\ \dot{y}_2 &= v_2 \end{aligned}$$

equivalent to

$$\begin{aligned} Y &= PV \\ P(s) &= \frac{1}{s} I_2 \end{aligned}$$

where $y_1 = S$ and $y_2 = VCO_2$ are the components of y defined in (3.3), $Y = [Y_1, Y_2]^T$ and $V = [V_1, V_2]^T$ where Y_i and V_i are respectively the Laplace transforms of y_i and v_i.

Then the matrices A and $b(\xi)$ defined in the Subsection 6.2.2 are

$$A = I_2$$

and

$$b(\xi) = \begin{bmatrix} L_1(\xi) \\ L_2(\xi) \end{bmatrix}$$

where

$$\begin{aligned} L_1(\xi) &:= -\nu(E,S)Tr \\ L_2(\xi) &:= -\frac{Tr^2 \nu(E,S)(K_S + 0.464 K_{SI} \alpha_S S^2 E^{\alpha_S - 1})}{2.17(S + K_S + K_{SI} S E^{\alpha_S})^2} \\ &\quad + \frac{\nu(E,S)}{2.17}[c_1 \nu(N) X (c_2 - f(S)) + g(E) Tr] \end{aligned}$$

because

$$\begin{aligned} [L_f^0 h_1(\xi), L_f^1 h_1(\xi)] &= [S, L_1(\xi)] \\ \left[L_f^0 h_2(\xi), L_f^1 h_2(\xi)\right] &= [VCO_2, L_2(\xi)] \\ [a_{10}, a_{11}] &= [0, 1] \\ [a_{20}, a_{21}] &= [0, 1]. \end{aligned}$$

Using the equation (6.8), we have the transformation searched and new control constrains defined in the Subsection 6.2.2,

$$\begin{aligned} u &= \Psi(\xi, v) \\ v_1^{min} &= L_1(\xi) + \beta_{11}(\xi) Q_1^{min} + \beta_{12}(\xi) Q_2^{max} \\ v_1^{max} &= L_1(\xi) + \beta_{11}(\xi) Q_1^{max} + \beta_{12}(\xi) Q_2^{min} \\ v_2^{min} &= L_2(\xi) + \beta_{21}(\xi) Q_1^{max} + \beta_{22}(\xi) Q_2^{max} \\ v_2^{max} &= L_2(\xi) + \beta_{21}(\xi) Q_1^{min} + \beta_{22}(\xi) Q_2^{min} \end{aligned}$$

where

$$\Psi(\xi, v) = \beta^{-1}(\xi)(v - b(\xi)).$$

It is because β_{11} is positive and $\beta_{12}, \beta_{21}, \beta_{22}$ are negative.

7.3 Anti-windup for linear system

We propose the following IMC controller K for the previous linear system:

$$K = \lambda_0 I_2 \quad \lambda_0 > 0.$$

Using the filter $F(s) = \frac{(\lambda_0+s)}{\lambda_0} I_2$ and the anti-windup technique in Section 4.3, the anti-windup controllers K_1 and K_2 are:

$$\begin{aligned} K_1 &= I_2 \\ K_2 &= (\tfrac{1}{\lambda_0} - 1) I_2 \end{aligned}$$

7.4 Close loop

We present the implicit mathematical solution using the anti-windup linearizing scheme. Before to do that, it is neccesary to follow a sequence of steps: linear scheme without constraints, nonlinear scheme without constraints, and finally, the anti-windup linearizing scheme with constraints.

a) Linear system without constraints:

$$\begin{aligned} Y &= PV \\ V &= K(Y^* - Y) \end{aligned}$$

The implicit solution of the control problem of this item is:

$$\begin{aligned} \dot{S} &= v_1 \\ V\dot{C}O_2 &= v_2 \\ v_1 &= \lambda_0(S^* - S) \\ v_2 &= \lambda_0(VCO_2^* - VCO_2) \end{aligned}$$

b) Nonlinear system without constraints:

$$\begin{aligned} y &= system(u) \\ u &= \Psi(x, v) \\ V &= K(Y^* - Y) \end{aligned}$$

The implicit solution of the control problem of this item is:

$$\begin{aligned} \frac{dN}{dt} &= -\mu(N)X + (N_{in} - N)\frac{Q_1}{V} + (N_{add} - N)\frac{Q_2}{V} \\ \frac{dX}{dt} &= \mu(N)f(S)X - X\frac{Q_1}{V} - X\frac{Q_2}{V} \\ \frac{dTr}{dt} &= c_1\mu(N)X(c_2 - f(S)) + g(E)Tr - Tr\frac{Q_1}{V} - Tr\frac{Q_2}{V} \\ \frac{dS}{dt} &= -\nu(E,S)Tr + (S_{in} - S)\frac{Q_1}{V} - S\frac{Q_2}{V} \\ \frac{dE}{dt} &= 0.464\nu(E,S)Tr + (E_{in} - E)\frac{Q_1}{V} - E\frac{Q_2}{V} \end{aligned}$$

$$
\begin{aligned}
VCO_2 &= \tfrac{1}{2.17}\nu(E,S)Tr \\
[Q_1, Q_2] &= \Psi(v,\xi) \\
v_1 &= \lambda_0(S^* - S) \\
v_2 &= \lambda_0(VCO_2^* - VCO_2)
\end{aligned}
$$

c) Anti-windup linearizing system with constraints:

$$
\begin{aligned}
y &= system(\tilde{u}) \\
\tilde{u} &= sat(u) \\
u &= \Psi(\tilde{v}) \\
V &= K_1(Y^* - Y) - K_2\tilde{V} \\
\tilde{v} &= sat(v,x)
\end{aligned}
$$

The implicit solution of the control pronlem of this item is:

$$
\begin{aligned}
\frac{dN}{dt} &= -\mu(N)X + (N_{in} - N)\frac{\tilde{Q}_1}{V} + (N_{add} - N)\frac{\tilde{Q}_2}{V} \\
\frac{dX}{dt} &= \mu(N)f(S)X - X\frac{\tilde{Q}_1}{V} - X\frac{\tilde{Q}_2}{V} \\
\frac{dTr}{dt} &= c_1\mu(N)X(c_2 - f(S)) + g(E)Tr - Tr\frac{\tilde{Q}_1}{V} - Tr\frac{\tilde{Q}_2}{V} \\
\frac{dS}{dt} &= -\nu(E,S)Tr + (S_{in} - S)\frac{\tilde{Q}_1}{V} - S\frac{\tilde{Q}_2}{V} \\
\frac{dE}{dt} &= 0.464\nu(E,S)Tr + (E_{in} - E)\frac{\tilde{Q}_1}{V} - E\frac{\tilde{Q}_2}{V} \\
VCO_2 &= \tfrac{1}{2.17}\nu(E,S)Tr \\
\tilde{Q}_1 &= sat(Q_1, Q_1^{min}, Q_1^{max}) \\
\tilde{Q}_2 &= sat(Q_2, Q_2^{min}, Q_2^{max}) \\
[Q_1, Q_2] &= \Psi(v,\xi) \\
v_1 &= S^* - S - (\tfrac{1}{\lambda_0} - 1)\tilde{v}_1 \\
v_2 &= VCO_2^* - VCO_2 - (\frac{1}{\lambda_0} - 1)\tilde{v}_2 \\
\tilde{v}_1 &= sat(v_1, v_1^{min}, v_1^{max}) \\
\tilde{v}_2 &= sat(v_2, v_2^{min}, v_2^{max})
\end{aligned}
$$

7.5 Simulation results

The following graphics describe the input and output of the control problems of the previous subsection solved in python language using the following values: the initial conditions of the system are $\xi(0) = [3, 0.7, 0.01, 200]^T$, the input concentrations are $[N_{in}, S_{in}, E_{in}, N_{add}] = [0.7, 200, 0, 0.063]$, the volume of the tank is $V = 1$, the control constraints are $[Q_1^{min}, Q_1^{max}] = [0, 0.1]$ and $[Q_2^{min}, Q_2^{max}] = [0, 0.07]$, the constant λ_0 is equal to 0.05 and finally the set-points are $[S^*, VCO_2^*] = [90, 1]$.

a) Linear system without constraints:

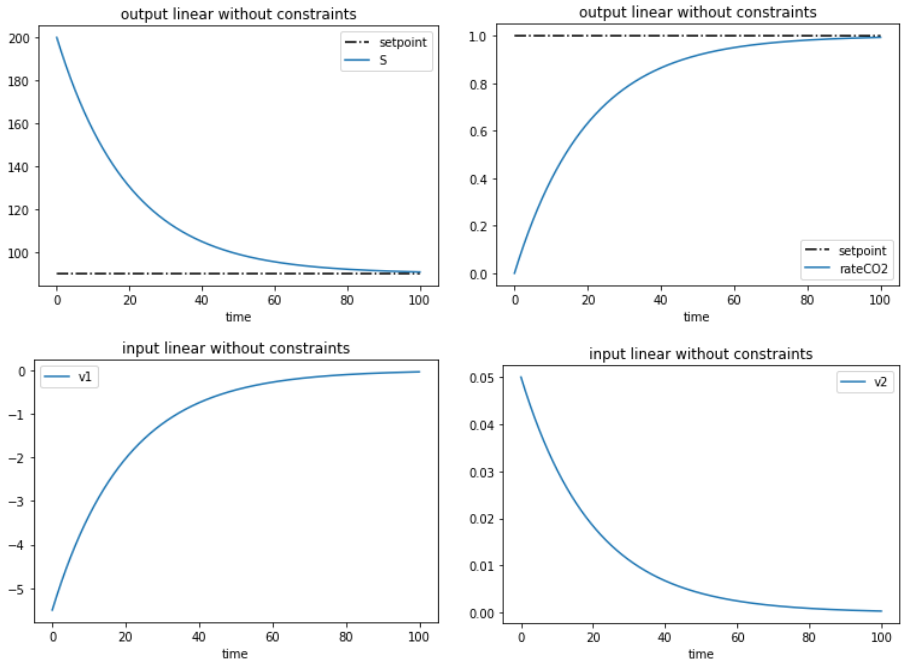


Figure 7.1: Linear solution without constraints.

b) Linearizing nonlinear system without constraints:

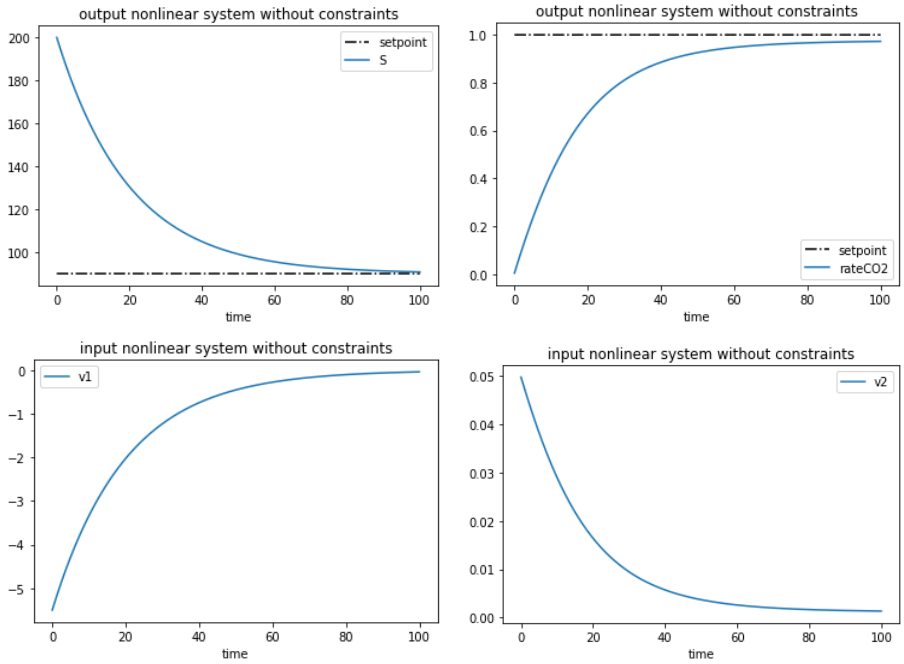


Figure 7.2: Linearizing nonlinear system without constraints.

c) Anti-windup linearizing nonlinear system with constraints:

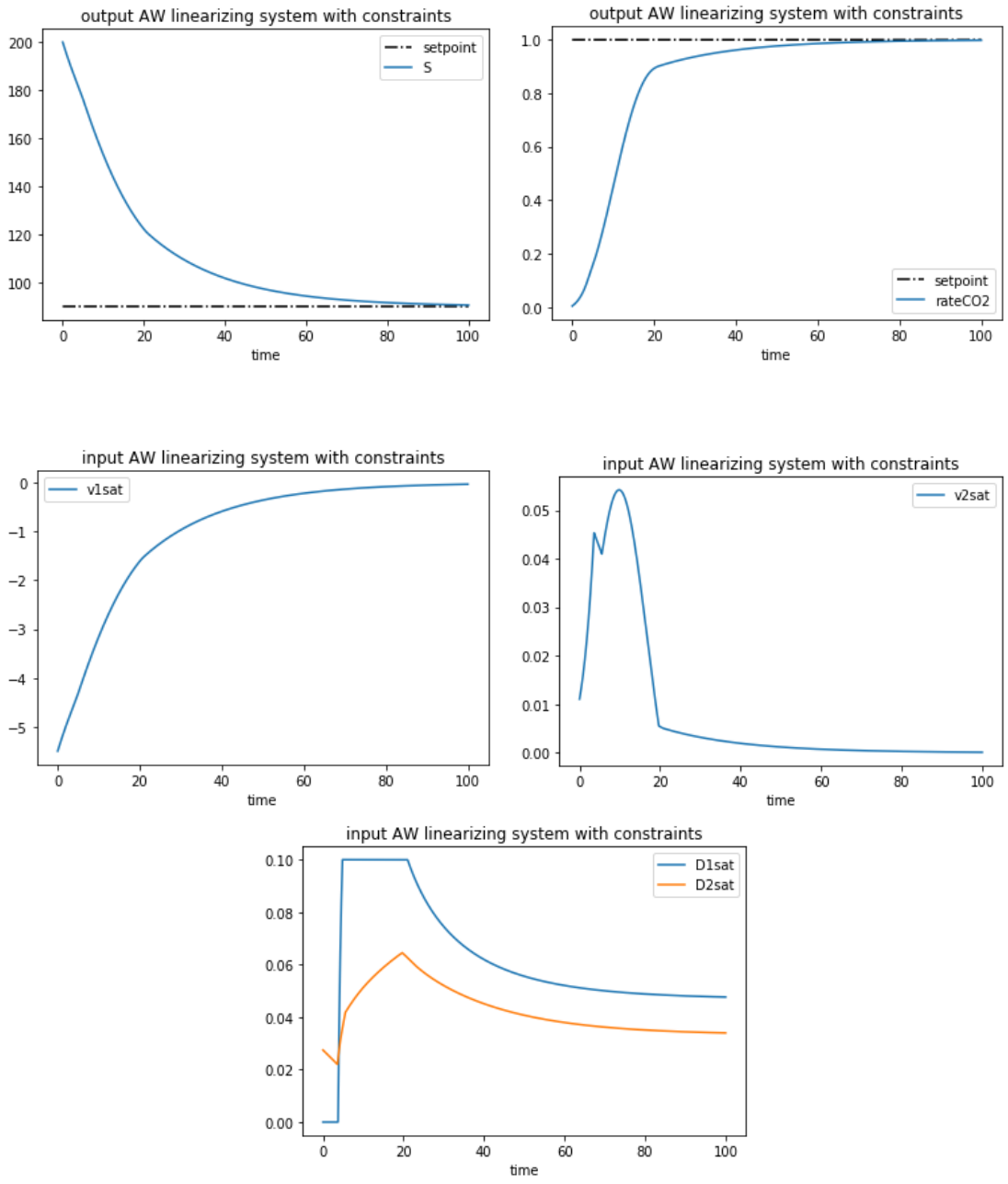


Figure 7.3: Anti-windup linearizing nonlinear system with constraints.

Conclusions

In Subsection 1.1.2 we have presented and described the mathematical model of the wine fermentation process with an oenological condition (addition of nitrogen) made by Malherbe in [2]. Simulation results of this model using python language and presented in Section 1.3 show us that the model is not good enough because the yeasts have a logistic behaviour. Because this limitation, we present a new mathematical model based in the Malherbe's model representing the behaviour of the wine fermentation with the same oenological condition. The simulation results of this new model presented in Section 2.3 show us that the model has a better approximation of behaviour regarding the wine fermentation.

In Chapter 7 we have achieved to apply the control theory to the wine fermentation process with an oenological condition, linearizing and using the anti-windup thecnique for linear systems presented in [6] and [4].

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List of Symbols

Symbol	Description	Page
$CO_2(t)$	carbon dioxide released (g/l)	11
$S(t)$	glucose concentration (g/l)	12
S_{init}	initial glucose concentration (g/l)	12
$N(t)$	nitrogen concentration (g/l)	12
N_{init}	initial nitrogen concentration (g/l)	12
$N_{add}(t)$	concentration of added nitrogen (g/l)	12
$X(t)$	cell population in the tank or yeast concentration $(cell/l)$	12
X_{init}	initial yeast concentration $(cell/l)$	12
$X_{max}(init)$	maximum population size during the stationary phase $(cell/l)$	12
$E(t)$	ethanol concentration (g/l)	12
E_{init}	initial ethanol concentration (g/l)	12
ν_{ST}	function describing the active transport of sugar per a glucose transporter (g/h)	12
N_{ST}	number of glucose transporters in a cell	12
K_S	affinity constant of glucose transporters (g/l)	12
K_{SI}	constant for the inhibition of glucose transporters by ethanol (l/g)	12
ν_N	function describing the active transport of nitrogen per a cell (g/h)	13
K_N	affinity constant of yeast cells for nitrogen transport (g/l)	13
K_{NI}	constant for the inhibition by ethanol of nitrogen transport in yeast cells (l/g)	13
T	temperature (C)	12
k_1, k_2, k_3	functions depending on temperature	12,13
α_S, α_N	constants	12,13
$Tr(t)$	number of glucose transporters in a cell	21
Tr_{init}	initial number of glucose transporters in a cell	21
$\mu_{max}, k_n, a, b, c, l, m, n$	new model parameters	21
$\|\| \quad \|\|_2$	Euclidean norm	23

$Q_1(t), Q_2(t)$	influent flow rates	29
Q_1^{min}, Q_2^{min}	minimum influent flow rates	29
Q_1^{max}, Q_2^{max}	maximum influent flow rates	29
$Q(t)$	efluent flow rate	29
$N_{init}, S_{init}, E_{init}$	chemical concentrations in the influent	29
V	volume	29
$V\mathrm{CO}_2(t)$	rate production or carbon dioxide rate	30
$\xi(t)$	state variable	30
$u(t)$	input or control	30
$y(t)$	output	30
y^*	known desirable constant value or setpoint	31
MIMO	multi-input multi-output	31
$u^{wc}(t)$	control for linear control systems without constraints	31
$y^{wc}(t)$	output for linear control systems without constraints	31
$[\mathcal{L}(f)](s)$	Laplace transforms of the real function f	31
$\mathcal{L}^{-1}[F](t)$	inverse Laplace transforms of the function F	31
$P(s)$	linear plant	32
$(p * u)(t)$	convolution transform of the functions p and u	32
$sat(u)$	saturation operator of u	33
$\tilde{u}(t)$	saturation of the control u	34
$K(s)$	controller without anti-windup	33
$K_1(s), K_2(s)$	controllers with anti-windup	35