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# Coulomb and Even-Odd Effects in Cold and Super-asymmetric Fragmentation for Thermal Neutron Induced Fission of $^{235}\text{U}$

M. Montoya

*Universidad Nacional de Ingeniería, Av. Túpac Amaru 210, Rímac, Lima, Peru.*

mmontoya@uni.edu.pe

**Abstract.** Even-odd effects of the maximal total kinetic energy ( $K_{\max}$ ) as a function of charge ( $Z$ ) and mass ( $A$ ) of fragments from thermal neutron induced fission of actinides are questioned by other authors. In this work, visiting old results on thermal neutron induced fission of  $^{235}\text{U}$ , those even-odd effects are reconfirmed. The cases seeming to contradict even-odd effects are interpreted with the Coulomb effect hypothesis. According to Coulomb effect hypothesis,  $K_{\max}$  is equal to the Coulomb interaction energy of the most compact scission configuration. As a consequence, between two isobaric charge splits with similar  $Q$ -values, the more asymmetrical one will get the more compact scission configuration and then it will reach the higher  $K_{\max}$ -value. In some cases, the more asymmetrical charge split corresponds, by coincidence, to an odd charge split; consequently its higher  $K_{\max}$ -value may be misinterpreted as anti-even-odd effect. Another experimental result reported in the literature is the increasing of even-odd effects on charge distribution on the more asymmetrical fragmentations region. In this region, the difference between  $K_{\max}$  and  $Q$ -values increases with asymmetry, which means that the corresponding scission configuration needs higher total deformation energy to occur. Higher deformation energy of the fragments implies lower free energy to break nucleon pairs. Consequently, in the asymmetric fragmentation region, the even-odd effects of the distribution of proton number and neutron number must increase with asymmetry.

## INTRODUCTION

The even-odd effects on the maximal value of total kinetic energy, as a function of charge and mass of fragments from thermal neutron induced fission of actinides, are properties that have generated controversy. In order to describe the nature of that controversy is useful to recall some definitions related to those even-odd effects [1]. Let be a fissile nucleus with charge  $Z_f$  and mass  $A_f$  that splits in a light fragment with  $Z_L$  protons,  $N_L$  neutrons (number of nucleons  $A_L = Z_L + N_L$ ) and a heavy fragment with  $Z_H$  protons,  $N_H$  neutrons (number of nucleons  $A_H = Z_H + N_H$ ), respectively. These numbers obey the following relations:

$$Z_f = Z_L + Z_H$$

and

$$A_f = A_L + A_H.$$

Then, to identify the two complementary fragments from a fission event it is enough to know the charge ( $Z$ ) and the proton number ( $N$ ) or the mass number ( $A$ ) of the light fragment.

After scission, the light and heavy fragments acquire kinetic energies  $K_L$ ,  $K_H$  and excitation energies  $X_L$ ,  $X_H$ , respectively. Thus, the total kinetic energy ( $K$ ) and the total excitation energy ( $X$ ) are given by the relations

$$K = K_L + K_H$$

and

$$X = X_L + X_H,$$

respectively. These quantities are limited by the energy balance equation:

$$Q = K + X,$$

where  $Q$  is the available energy of the reaction. See Ref. [2].

At the scission configuration, available energy is spent into deformation energy ( $D$ ), Coulomb interaction energy ( $C$ ) and free energy ( $F$ ):

$$Q = C + D + F.$$

Free energy is partitioned into intrinsic energy ( $X^*$ ) and total pre-scission energy of fragments ( $K_S$ ):

$$F = X^* + K_S.$$

See Ref. [3].

Preference for even proton numbers in the fission fragments has been well established [1], leading to the definition of the even-odd effect of charge distribution ( $\delta Z$ ):

$$\delta Z = \frac{Y_e^Z - Y_o^Z}{Y_e^Z + Y_o^Z},$$

where  $Y_e^Z$  y  $Y_o^Z$  are the yields of fragments with even and odd proton numbers, respectively. Even-odd effect in the distribution neutron number ( $\delta N$ ) and nucleon number ( $\delta A$ ), respectively, are defined in a similar way.

One assumes that, for a given fragmentation corresponding to proton and mass numbers  $Z$  and  $A$ , respectively, the maximum total kinetic energy ( $K_{\max}$ ) is reached by a configuration that at scission acquires a maximum Coulomb interaction energy ( $C_{\max}$ ) and a minimum total deformation energy ( $D_{\min}$ ), limited by the equation

$$Q = C_{\max} + D_{\min}.$$

Because Coulomb repulsion between fragments, Coulomb interaction energy becomes total kinetic energy, so that:

$$K_{\max} = C_{\max} = Q - D_{\min}.$$

Let be  $A$  an odd nucleon number of the light fragment, the local even-odd effect in the maximum  $Q$ -value ( $Q_{\max}^A$ ) as a function of mass is defined as

$$\delta_A Q_{\max} = \frac{Q_{\max}^{A-1} + Q_{\max}^{A+1}}{2} - Q_{\max}^A.$$

In average  $\delta_A Q_{\max}$  is positive. Local even-odd effects in  $Q_{\max}$  as a function of proton number ( $\delta_Z Q_{\max}$ ) and neutron number ( $\delta_N Q_{\max}$ ), respectively, are defined in a similar way.

Taking an odd mass number, one may also define local even-odd effect in the maximum total kinetic energy as a function of mass:

$$\delta_A K_{\max} = \frac{K_{\max}^{A-1} + K_{\max}^{A+1}}{2} - K_{\max}^A.$$

Similarly,  $\delta_Z K_{\max}$  and  $\delta_N K_{\max}$  are defined as even-odd effects in the maximum total kinetic energy as a function of proton and neutron numbers, respectively.

Because even-odd effects on proton and neutron distribution, respectively, increase with fragment kinetic energy [1], positive values of  $\delta_A K_{\max}$ ,  $\delta_N K_{\max}$  and  $\delta_Z K_{\max}$  are expected.

In 1981 C. Signarbieux *et al.* found the evidence of the existence of cold fission, in which the excitation energy is not enough for the fragments to emit neutrons. Contrary to expected, they did not find a significant even-odd effect in the distribution of the nucleon numbers ( $\delta A \cong 0$ ) [4].

In 1981, M. Montoya deduced [5, 6] that, if one assumes that no more than one nucleon pair is broken,

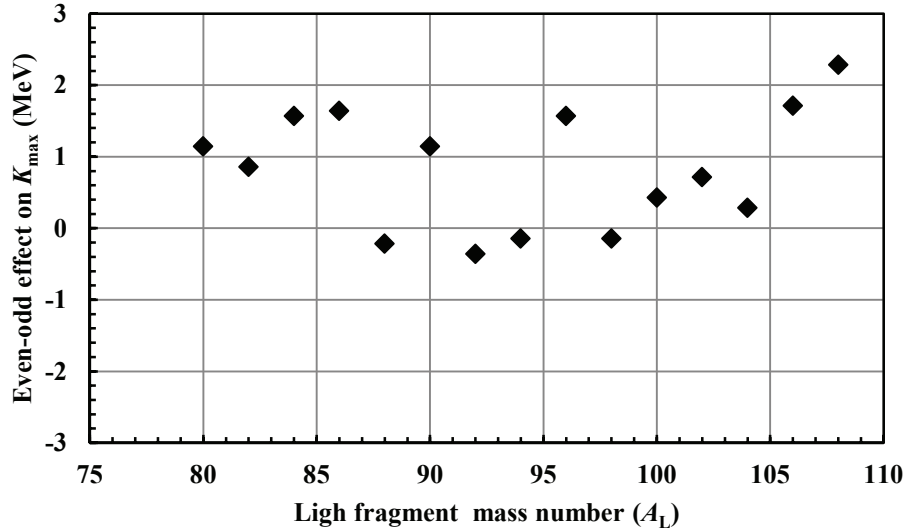
$$\delta A = \delta Z + \delta N - 1,$$

which was confirmed by H. Nifenecker [7]. That relation shows that there is no contradiction between a low even-odd effect in  $A$  distribution and high even-odd effects in  $Z$  and  $N$  distributions, respectively.

In 1991, based on data communicated by C. Signarbieux *et al.*, F. Gönnerwein and B. Börsig show that the minimum excitation energy,

$$X_{\min} = Q_{K_{\max}} - K_{\max},$$

where ( $Q_{K_{\max}}$ ) is the  $Q$ -value corresponding to the charge that maximizes the total kinetic energy, is lower for the odd than for the even proton numbers [8]. This result encourages research on even-odd effects in fission, which leads to review existent results and interpretations about even-odd effects in the distribution of mass, charge and maximum total kinetic energy of fragments.



**FIGURE 1.** Even-odd effect on the maximum total kinetic energy of fragments from thermal neutron induced fission of  $^{235}\text{U}$ , calculated with data from Ref. [9].

In this work, in order to analyze more carefully even-odd effects of  $K_{\max}$  as a function of  $A$ , data on thermal neutron induced fission of  $^{235}\text{U}$  is revisited.

### EVEN-ODD EFFECTS IN THE MAXIMUM TOTAL KINETIC ENERGY

Taking into account the  $3 \times 10^6$  events from thermal neutron induced fission of  $^{235}\text{U}$ , obtained in 1981 by C. Signarbieux *et al.* [4], in 1984 M. Montoya presents the curve of threshold values for the 10 events with the highest total kinetic energy values as a function of light fragment mass [9]. These threshold values are assumed to correspond to the maximum total kinetic energy for each isobaric fragmentation. In Fig. 1 one can see the calculated the values of even-odd effect of  $K_{\max}$  as a function of mass obtained with that data. The average of  $\delta_A K_{\max}$  is 0.83 MeV.

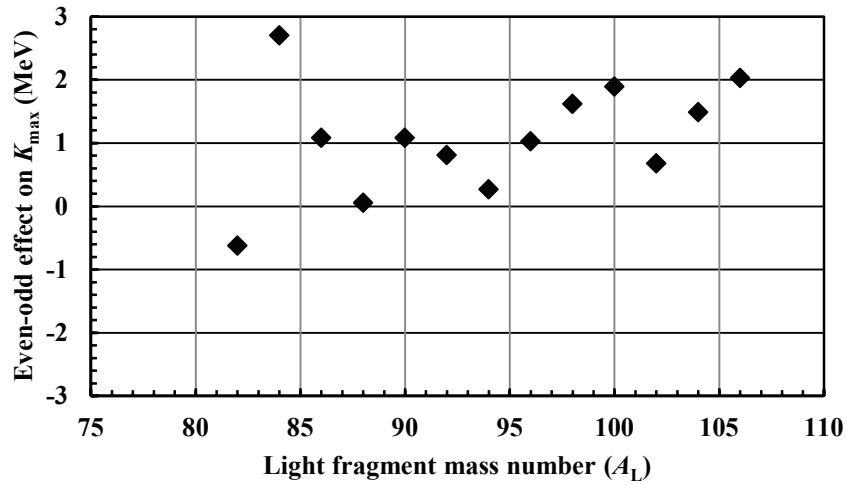


FIGURE 2. Even-odd effect on the maximum total kinetic energy of fragments from thermal neutron induced fission of  $^{235}\text{U}$ , calculated with data from Ref. [10].

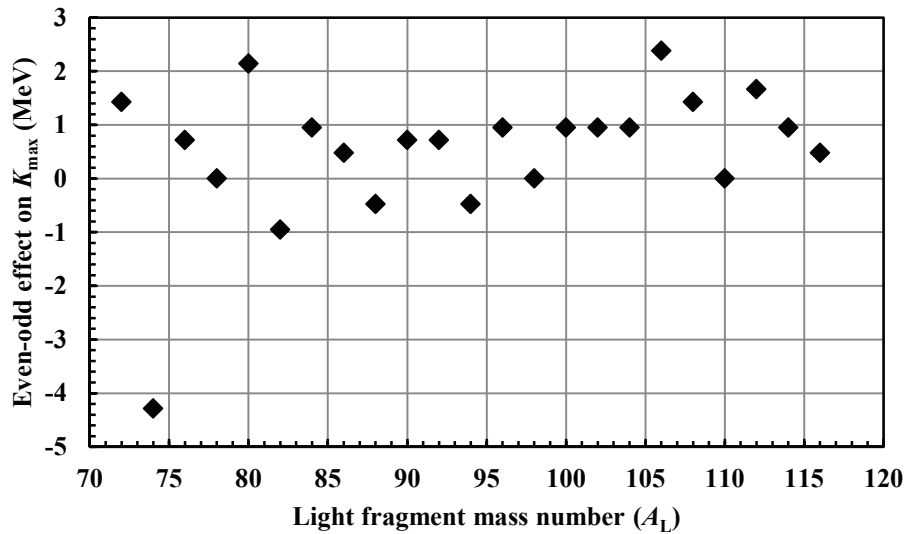


FIGURE 3. Even-odd effect on the maximum total kinetic energy of fragments from thermal neutron induced fission of  $^{235}\text{U}$ , calculated with data from Ref. [8].

In 1986 G. Simon *et al.* measured maximum total kinetic energy as a function of charge and mass of fragments from thermal neutron fission of  $^{235}\text{U}$  [10]. Using their data it results an average even-odd effect in the maximum total kinetic energy as a function of mass ( $\delta_A K_{\max}$ ) equal to 1.08 MeV. See Fig. 2. The  $K$ -values corresponding to the groups of the neighboring masses (92, 93 y 94), (98, 99, 100) and (102, 103, 104) are maximized by charges (36, 37, 38), (38, 39, 40) and (40, 41, 42), respectively. For these groups, the average of  $\delta_Z K_{\max}$  is = 0.66 MeV.

Based on data communicated by C. Signarbieux, F. Gönnerwein and B. Börsig present the maximum total kinetic energy as a function of mass of fragments from thermal neutron induced fission of  $^{235}\text{U}$ . The charges that maximize the total kinetic energy and the charges that maximize the available energy are compared [8]. The average even-odd effect of the maximum total kinetic energy value as a function of the mass ( $\delta_A K_{\max}$ ) is 0.5 MeV. This value is lower than the corresponding to the two cases presented in Figs. 1 and 2. This is due to the value of approximately  $-4$  MeV corresponding to  $A_L = 74$ . If one takes three neighboring masses, corresponding to three different charges, in average it results  $\delta_Z K_{\max} = 0.8$  MeV. If one uses the mass table of Ref [11] the average of odd-even effect on the maximum available energy as a function of mass is 1.6 MeV.

Those results are consistent with the hypothesis of a positive odd-even effect in  $K_{\max}$ -values. If one assumes that

$$Q_{K_{\max}} = C_{\max} + D_{\min},$$

$$\delta_Z Q = Q^e - Q^o > 0,$$

and

$$D^e(D) = D^o(D),$$

where  $D$  is the deformation of the scission configuration. See Figs. 4a and 4b.

From those relations, if one assumes that the neighboring even and odd charge fragments have the same stiffness, i.e. same energy for same deformation, one can say that

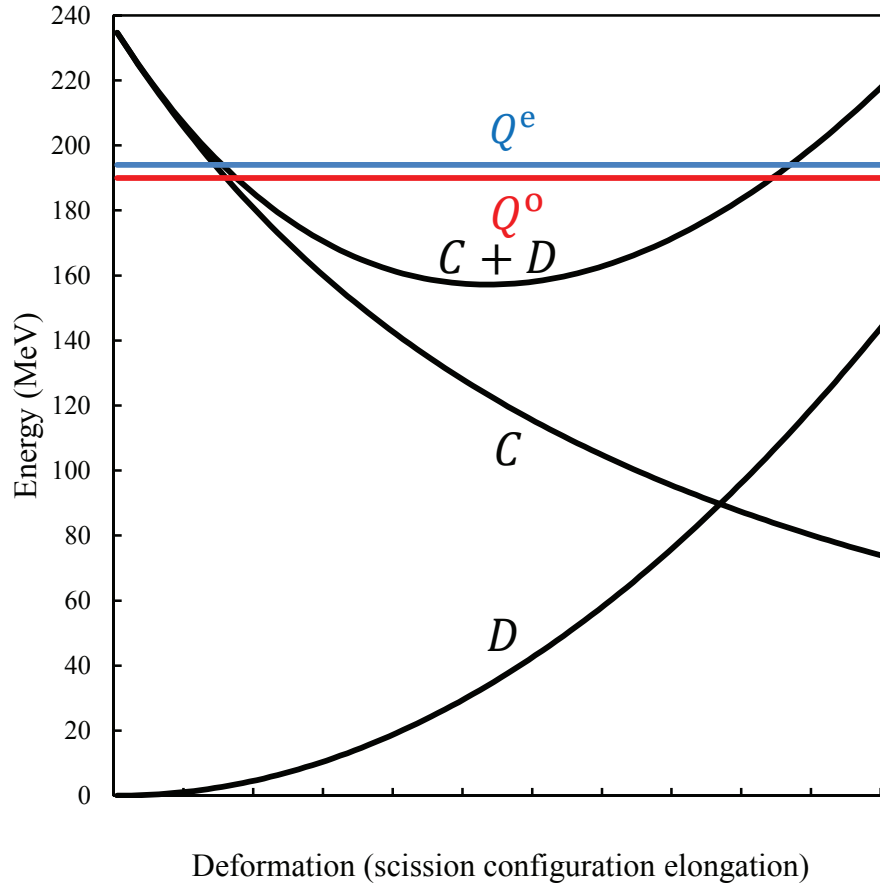
$$\delta_Z C_{\max} = \delta_Z K_{\max} = \delta_Z Q_{K_{\max}} - \delta_Z D_{\min} > \delta_Z Q_{K_{\max}}.$$

Similar relations may be deduced for even-odd effect on  $C_{\max}$  as a function of neutron or mass number, respectively.

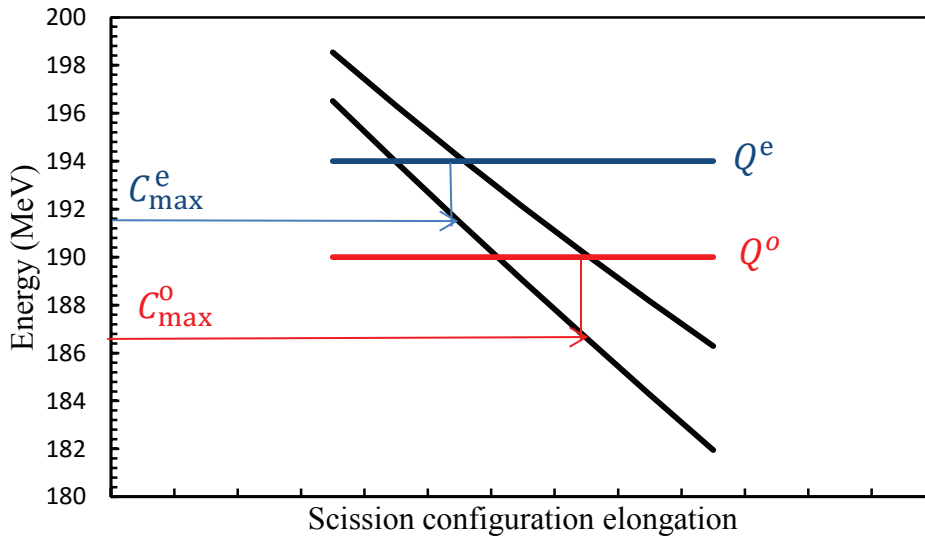
In Fig. 4a and 4b one can easily see that the even-odd effect on maximum value of  $K$  is higher than the even-odd effect on maximum value of  $Q$ .

In 2013, F. Gönnerwein shows that, for mass fragmentations 104/132, the kinetic energy associated to the charge fragmentation 41/51 reach the  $Q$ -value of the reaction, while the corresponding to the fragmentation 42/50 reaches a total maximum kinetic energy below 3 MeV the corresponding  $Q$ -value [12]. Gönnerwein suggests that this is due to the fact the charge split 41/51 corresponds to odd fragment charges. However, we should note that charge fragmentation 41/51 is more asymmetric than the 42/50 fragmentation. Therefore that result is also consistent with the Coulomb effect: for neighboring masses with similar values of energy available, the more asymmetric fragmentation reaches the higher values of total kinetic energy [13, 14].

In general, results presented by F. Gönnerwein show that the charges that maximize the total kinetic energy are the same as the charges that maximize the available energy. Exceptions occur for the masses 88, 97, 98 and 99, whose charges that maximize the available energy are 36, 39, 40 and 40, respectively, whereas the corresponding charges that maximize the total kinetic energy are 35 (<36), 38 (<39), 38 (<40) and 39 (<40), respectively. These results are also consistent with the hypothesis of the Coulomb effect [13, 14].

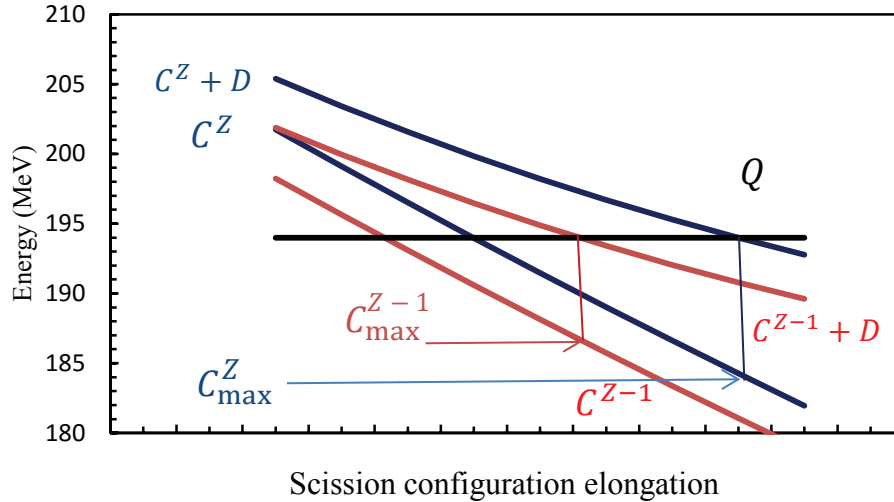


**FIGURE 4a.** Schematic representation of Coulomb energy ( $C$ ), deformation energy ( $D$ ) and total potential energy ( $C + D$ ) of a scission configuration from thermal neutron induced fission of actinides. Available energy for even charge ( $Q^e$ ) and odd charge splits ( $Q^o$ ).



**FIGURE 4b.** Amplifying area shown in Figure 4a. One can see that  $C_{\max}^e - C_{\max}^o > Q_{\max}^e - Q_{\max}^o$ . In other words, even-odd effect on  $C_{\max}$  must be higher than even-odd effect on  $Q_{\max}$ .

To better explain the Coulomb effect [13, 14], one can see Fig. 5. For two isobaric splits or neighboring mass splits with same  $Q$ -value, with light fragment charges  $Z$  and  $Z - 1$ , respectively, the more asymmetric charge split reaches lower Coulomb interaction energy, which permits it to get more compact configuration and, consequently higher maximum Coulomb interaction energy and finally higher total kinetic energy.



**FIGURE 5.** Schematic representation of Coulomb effect on maximum total kinetic energy of fragments from low energy fission of actinides.  $Q$ -value (horizontal black line) intersects the total potential energy ( $C + D$ ) curve for the most compact scission configuration, which correspond to maximum Coulomb interaction energy. The more asymmetric charge split (red curves) reaches the higher maximum Coulomb energy than the more symmetric charge split (blue curves) does.

## COULOMB AND EVEN-ODD EFFECTS IN SUPER-ASYMMETRIC FRAGMENTATIONS

In 1989, J. L. Sida *et al.* note that in the region of super-asymmetric charge fragmentations, even-odd effects in charge and neutron number yields, respectively, increase with charge asymmetry [15]. This result is consistent with the hypothesis of Coulomb effect. Indeed, if one assumes a scission configuration with spherical fragments whose surfaces, the Coulomb interaction energy is higher than the corresponding  $Q$ -value. Therefore at scission point the fragments must be deformed so that the energy of Coulomb interaction is lower than the available energy [13, 14]. For masses lower than 104 the Coulomb interaction energy curve separates from the  $Q$ -value to the extent that the fragmentation is asymmetric. This implies that the higher the asymmetry, the higher the deformation energy of the fission fragments must be to make the fission possible. Therefore, the process has lower free energy,

$$F = Q - C - D.$$

A lower free energy implies a lower probability to break pairs of nucleons, therefore a higher even-odd effect in yields of charge. This is precisely what is experimentally observed.

## CONCLUSION

Contrary to suggestion of Gönnerwein [12], analyzing three sets of old experimental results about thermal neutron induced fission of  $^{235}\text{U}$ , even odd effects on maximal value of total kinetic energy of fragments as a function of  $A$  and  $Z$ , respectively, is confirmed.



Those experimental results about cold fission suggest that the most compact scission configuration is reached without intrinsic excitation energy to break nucleon pairs, which implies the existence of even-odd effect in maximal value of total kinetic energy of fragments.

Super-asymmetrical fragmentations need high deformation energy to fulfill the energy balance condition; which implies low intrinsic excitation energy and, consequently, a high even-odd effect of charge and neutron number distributions, respectively.

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